PRICE INDEXES
AND
QUALITY CHANGES

BY

ERLAND v. HOFSTEN

STOCKHOLM

1952

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QUALITY CHANGES

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ERLAND v. HOFSTEN
FIL. LIC.

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Preface

The origin of this investigation can be traced back to a conversation which I had in April 1948 with Mr. Edward D. Hollander, Chief of the Division of Prices and Cost of Living at the Bureau of Labor Statistics, Washington. A year earlier I had become chief of the statistical section of the Social Welfare Board, which is responsible for the calculation of the Swedish cost-of-living index, and I naturally wished to take advantage of my visit to Washington to study the methods used for the corresponding U. S. index. In the course of that conversation I realized that the American and Swedish index calculations did not make allowance for quality changes in the same way, and that this difference might very well considerably affect the final result. The problem called for treatment.

My original intention was to attack the problem only empirically. Despite this, I gradually felt it essential to widen the scope of the study, with the result that the treatise now presented is somewhat theoretical in nature. I have endeavoured to keep in mind, however, that the main purpose was to lay a foundation upon which actual practice could be soundly based. Consequently, the theoretical arguments have not been pursued very far. Many of the problems treated require, therefore, a stricter mathematical analysis.

It has been my privilege during this work to be in contact with Professor Herman Wold, Upsala, and to participate in this graduate statistical seminar, where preliminary drafts of my manuscript have been discussed on several occasions. Without the help of Professor Wold's experience, and his positive criticism, it could not have been possible to finish this study. A member of the seminar whom I should especially wish to mention as having influenced my work at an early stage is Mr. Gunnar Eklund.

I have had the opportunity of discussing some of the problems treated with Professor R. G. D. Allen, London, Professor Ragnar Frisch, Oslo, Dr. R. C. Geary, Dublin, and Dr. Hans Staehle, Geneva. It goes without saying that these conversations have been of great benefit to me.

Among others who have contributed with imaginative suggestions I should like to mention Mr. Krister Wickman, Stockholm. My sincere thanks are also due to Dr. Sven Moberg and Mr. Erik Ruist for a number of valuable improvements.

Of course, none of the persons mentioned above must in any way be held responsible for any shortcomings in this work. For these, the author alone is answerable.

The text was written in English, and Dr. Peter Whittle, Upsala, has helped me with the revision of the language.

Finally, I should like to express my gratitude to the Swedish Social Science Research Council for financial assistance.

STOCKHOLM

February 29, 1952 *E. v. H.*

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Chapter 1.—Price Index Problems in Theory

The literature on index numbers is very rich.[1] As an introduction to the treatment of the specific price index problem, to which this monograph is devoted, this chapter will give an account of the more important lines of thought in the theoretical index discussion, including some new remarks.

The problems discussed in this book mainly refer to the computation of retail price indexes. However, much of what is said is valid also for wholesale price indexes. We consider to some extent the special problems which arise in the forming of place to place comparisons.

The classical index theory[2]

A price index aims at comparing the price level of two or more situations, i.e. points in time, or places. If there are only two situations, 0 and 1, and the prices of the goods in the two situations are designated $p_0', p_0'', p_0''' \ldots\ldots p_0^n$, and $p_1', p_1'', p_1''' \ldots\ldots p_1^n$, respectively, the index may be looked upon as some kind of average of the price quotients $\frac{p_1'}{p_0'}\ \frac{p_1''}{p_0''}$ etc. In practice the indexes are usually given as percentages, i.e. averages of the quotients $100\frac{p_1^i}{p_0^i}$.

The prices may be wholesale prices or retail prices, and only part of the market need be included, e.g. food prices only. In general it is assumed that prices are obtained for all the goods included in the group which the index should describe. It it also assumed that the

[1] In a (mimeographed) bibliography by William R. Buckland, London School of Economics, reference is made to 169 contributions between the years 1707 and 1946, in spite of its being limited to more important articles and books.

[2] The exposition given in this section to a certain extent follows the corresponding section in *Wold* (1952).

universe of goods is identical in the two situations, i.e that there is a price p_1^i corresponding to every price p_0^i and vice versa. The market is as a rule assumed free, i.e. rationing etc. is not taken into consideration.

Various averages may be constructed from the price quotients, such as the arithmetic, the harmonic, or the geometric mean (or the median etc.), and they may be weighted or unweighted. If the relative weights are designated w_i ($w_i \geq 0$ and $\Sigma w_i = 1$), the arithmetic, the harmonic, and the geometric means take the form:

$$(1.1)\quad I_{01}^{A} = \Sigma w_i \frac{p_1^i}{p_0^i}; \quad (1.2)\quad \frac{1}{I_{01}^{H}} = \sum \frac{w_i}{p_1^i/p_0^i}; \quad (1.3)\quad I_{01}^{G} = \Pi \left(\frac{p_1^i}{p_0^i}\right)^{w_i}$$

From these basic formulas a great number of specific index formulas may be formed. The weights may thus be defined by one of the formulas

$$(1.4)\qquad w_i = \frac{p_0 q}{\Sigma p_0 q}; \qquad\qquad (1.5)\qquad w_i = \frac{p_1 q}{\Sigma p_1 q}$$

where q designates quantities of the goods according to a certain budget. If the weights (1.4) are inserted in (1.1) or the weights (1.5) in (1.2), the same formula is obtained, viz.

$$(1.6)\qquad I_{01}^{A} = \frac{\Sigma p_1 q}{\Sigma p_0 q}$$

If the quantities refer to situation 0, the weights in formula (1.4) will signify the cost-weights in situation 0. Formula (1.6) then becomes Laspeyres' formula[3]:

$$(1.7)\qquad I_{01}^{L} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$$

If the quantities in (1.6) refer to situation 1, the weights in (1.5) will signify the cost-weights in situation 1, and formula (1.6) becomes Paasche's formula[4]:

$$(1.8)\qquad I_{01}^{P} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}$$

[3] Named after the German economist Étienne Laspeyres (1834–1913).
[4] Named after the German economist Hermann Paasche (1851–1925).

It should be noted that Paasche's formula is not obtained, if the cost-weights referring to situation 1 are inserted in formula (1.1). One then obtains

$$I_{01} = \frac{\sum \frac{p_1}{p_0} p_1 q_1}{\Sigma p_1 q_1} \tag{1.9}$$

which Fisher has called Palgrave's formula.[5]

To obtain Paasche's formula from the general arithmetic average formula, the weights must be defined as

$$w_i = \frac{\frac{p_0}{p_1}(p_1 q_1)}{\sum \frac{p_0}{p_1}(p_1 q_1)}$$

It is easily seen that if prices are higher in situation 1 than in situation 0, Palgrave's formula will give a higher result than Paasche's formula and vice versa. This difference may be of importance.

A great many other index formulas may be obtained from the general formulas (1.1) to (1.3). Of these only two will be mentioned here. The first of these is Edgeworth's formula, where the quantities in formula (1.6) represent the arithmetic average of the quantities in the two situations 0 and 1:

$$I_{01} = \frac{\Sigma p_1 \frac{1}{2}(q_0 + q_1)}{\Sigma p_0 \frac{1}{2}(q_0 + q_1)} = \frac{\Sigma p_1 (q_0 + q_1)}{\Sigma p_0 (q_0 + q_1)} \tag{1.10}$$

If the index comparison refers to two points in time, the quantities may also represent the consumption *between* 0 and 1. If these quantities are designated $\bar{q}$, formula (1.6) becomes:

$$I_{01} = \frac{\Sigma p_1 \bar{q}}{\Sigma p_0 \bar{q}} \tag{1.11}$$

This is the formula which is used in principle for the Swedish index calculations (cf. chapter 4). To start with price quotients are

[5] *Fisher* (1922), p. 466.

computed for the separate items, and the formula is consequently used in the following form:

$$I_{01} = \frac{\sum \frac{p_1}{p_0} \cdot \frac{p_0}{\bar{p}} \cdot \overline{pq}}{\sum \frac{p_0}{\bar{p}} \cdot \overline{pq}} \tag{1.12}$$

Formulas (1.10) and (1.11) will give approximately the same result, if the period covered is not too long. It should be noted, however, that if price changes are assumed to exercise an immediate influence upon the quantities consumed, formula (1.10) will give the same result if the price change occurs at the beginning or at the end of the period. The result obtained from formula (1.11) will on the other hand depend upon when the price changes have occurred during the period.

From a formal point of view all different index formulas must be considered as legitimate. In order to decide which formula shall be preferred, the classical index theory developed a number of index tests. The most well-known representative for this approach is Irving Fisher. It is, however, not possible to arrive at a formula, which satisfies all desirable criteria.[6]

Only three of Fisher's tests will be mentioned here, viz.

1) the factor reversal test,
2) the time (or place) reversal test, and
3) the circular test.

The factor reversal test implies that the quantity (volume) index, Q_{01}, which is obtained if all p's are substituted by q's and vice versa, after multiplication with the price index shall give the *value ratio*, i.e.

$$I_{01} \cdot Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \tag{1.13}$$

Fisher justifies this test in the following way: "(1) no reason can be given for employing a given formula for one of the two factors

[6] See *Fisher* (1922); cf. *Frisch* (1930), p. 405, *Frisch* (1936), pp. 6–7, and *Wald* (1937), p. 181.

which does not apply to the other, and, (2) such reversibility already applies to each pair of *individual* price and quantity ratios, and should, in all logic, apply to the index numbers which aim to represent them in the mass."[7]

The time reversal test implies that the index in one direction shall be the inverted value of the index in the other direction, i.e. $I_{01} \cdot I_{10} = 1$.

Fisher does not explicitly justify this test but considers its validity to be self-evident: "Either one of the two times may be taken as the 'base'. Will it make a difference which is chosen? Certainly it *ought* not . . ."[8]

The time reversal test is a special case of the more general *circular test* which presupposes a comparison between at least three times (or places). Then we shall have

$$I_{01} \cdot I_{12} = I_{02} \tag{1.14}$$

If the index refers to a number of subsequent points in time and a separate index is computed for every period, these separate indexes being multiplied by each other, *a chain index* is obtained

$$I_{0t} = I_{01} \cdot I_{12} \cdot I_{23} \ldots\ldots I_{t-1,t} \tag{1.15}$$

In general this index will give the same result as the direct comparison between 0 and t, only if the elementary index meets the circular test.

In the end Fisher gives up the circular test. He thus states that "I aim to show that the circular test is theoretically a mistaken one, that a necessary irreducible minimum of divergency from such fulfillment is entirely right and proper, and, therefore, that a *perfect* fulfillment of this so-called circular test should really be taken as proof that the formula which fulfills it is erroneous."[9]

If Laspeyres' formula is used, this test is not fulfilled, for then

$$I_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \quad \text{and} \quad I_{12} = \frac{\Sigma p_2 q_1}{\Sigma p_1 q_1}$$

[7] *Fisher* (1922), p. 75.
[8] *Fisher* (1922), p. 64.
[9] *Fisher* (1922), p. 271.

The product of these two expressions does not, however, give the same result as the direct comparison between 0 and 2, which gives

$$I_{02} = \frac{\Sigma p_2 q_0}{\Sigma p_0 q_0}$$

It is possible to determine under what conditions the value of $I_{01} \cdot I_{12}$ gives a different value from I_{02}. If Laspeyres' formula is used, one obtains that[1]

$$(1.16) \qquad D = \frac{I_{01} \cdot I_{12}}{I_{02}} = \frac{\dfrac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \cdot \dfrac{\Sigma p_2 q_1}{\Sigma p_1 q_1}}{\dfrac{\Sigma p_2 q_0}{\Sigma p_0 q_0}} = 1 + \frac{\sigma_x \sigma_y r_{xy}}{\bar{x} \cdot \bar{y}}$$

Here r_{xy} is the weighted correlation coefficient between $x = \frac{p_2}{p_1}$ and $y = \frac{q_1}{q_0}$, $\bar{x}$ and $\bar{y}$ the weighted averages of x and y, and σ_x and σ_y the weighted standard deviations of x and y. If then the quantity changes from 0 to 1 are positively correlated with the price changes from 1 to 2, the product $I_{01} \cdot I_{12}$ will give a higher result than the direct comparison, I_{02}.

It may be thought that this correlation should normally be positive and that consequently a chain Laspeyres comparison should give a higher result than a direct Laspeyres comparison. Frisch also says without qualification, that "Laspeyres' index will drift upwards."[2] This is obvious, if the links are shorter than a year and there are seasonal fluctuations of quantities and prices, the quantities being greatest when the prices are lowest and vice versa. This is the case with many food items, e.g. eggs, vegetables, fruit. For such items a chain index with short—weekly or monthly—links will be ever increasing, even if both quantities and prices every year return to the same values as in the corresponding season of the previous year. Törnqvist has demonstrated that a week-to-week index for food was after three years 20 per cent higher than a direct comparison.[3]

[1] Proof in *Bortkiewicz* (1924), p. 210. Cf. *Frisch* (1936), p. 8, and *Betänkande* (1943), p. 142. The difference between Laspeyres' and Paasche's formulas etc. may be analyzed in a similar way.

[2] *Frisch* (1936), p. 9.

[3] *Törnqvist* (1937), p. 77.

However, if the links are longer—a year or more—it is, as pointed out by Bortkiewicz[4], not possible to state that the correlation between quantities and prices will in general be such that the Laspeyres chain index gives a higher result than a direct Laspeyres comparison.

As pointed out by Frisch, the drifting of the chain index "must not be taken to mean that the direct index is right and the chain wrong. This cannot be decided by the above formal considerations."[5]

The theory so far described is general in the sense that it applies to all kinds of price index comparisons, i.e. both wholesale and retail prices, both indexes comprising all kinds of goods and those covering only a group of items as for instance food, petroleum products etc. It is characteristic of the earlier discussion on price indexes that it tried to solve the index problem in general and was not interested in any nearer specification of the aim of the index comparison.

It is thus striking to note that no really clarifying description of the function which a price index number should fulfil can be found in Fisher's book. He restricts himself to general phrases like the following one: a price index should indicate "the *average percentage change* of prices from one point of time to another."[6] The object is to find the formula or formulas, giving the 'best' index or to quote another of Fisher's expressions: "the purpose of any index number is to strike a 'fair average' of the price movements."[7]

In the more recent theory of index numbers greater importance is attached to the specific meaning of the index and the purpose which it should fulfil. There are two lines of development, the one being the indifference approach, developed by Konüs and others, and the other being the integral solution given by Divisia.

The indifference defined index

The main part of the theoretical literature of the last two decades on price index numbers is devoted to what Samuelson calls "the economic theory of index numbers" as contrasted to "the statistical

[4] *Bortkiewicz* (1924), p. 219.
[5] *Frisch* (1936), p. 9.
[6] *Fisher* (1922), p. 3.
[7] *Fisher* (1922), p. 43.

theory of index numbers."[8] Frisch makes a similar distinction between "the functional approach" and "the atomistic approach."[9]

The economic theory of index numbers is confined to 'cost of living' comparisons, i.e., only retail prices are considered. Most of the discussion is based on the assumption that the comparison is limited to two situations, and that it refers solely to one individual, whose total 'cost of living' is considered.

The most important contributor to the theory of the indifference defined index is Konüs.[1]

As an example of the definition of the object of index comparison, basic to this approach, we may quote Bowley's words: "What change in expenditure is necessary after a change of prices to obtain the same satisfaction as before?"[2] This formulation and several other similar ones are cited by Frisch.[3] Another definition is the one given by Schultz in his comments upon the English translation of Konüs' article: "The true index of the cost of living is the ratio of the money expenditures of an individual which secure for him the *same standard of living* or total utility in two situations which differ only in respect to prices."[4]

The further analysis is based on the theory of indifference levels. In the 'given' situation o the income of the consumer is equal to $\Sigma p_0 q_0$ (income = outlay; savings are assumed negligible). An indifference map with indifference loci, convex to the origin, is given, and it is further assumed that the consumer maximizes his utility function.

If, now, in situation 1, where prices are different, the consumer is given an income equal to $\Sigma p_1 q_0$, this would make it possible for him to maintain a consumption of exactly the same composition as in situation o. This consumption does, however, not necessarily represent a point of optimum satisfaction. By adapting himself to the

[8] *Samuelson* (1947), p. 146.
[9] *Frisch* (1936), p. 3.
[1] *Konüs* (1939). Konüs' article was originally written in Russian and published in 1924. Only part of his arguments were known to Western readers before the English translation was published in 1939. For further reference, see *Staehle* (1934), *Staehle* (1934–35), *Staehle* (1935–36), *Staehle* (1936–37), *Allen* (1933), *Allen* (1935–36), *Wald* (1937), and *Allen* (1949).
[2] *Bowley* (1928), p. 223.
[3] *Frisch* (1936), pp. 11–12.
[4] *Schultz* (1939), p. 2.

price changes, it is possible for the consumer in situation 1 to obtain the same satisfaction as that represented by $\Sigma p_0 q_0$ in situation 0 at a cost less than $\Sigma p_1 q_0$. If, then, in situation 1 the consumer is given an income equal to $\Sigma p_1 q_0$, it is possible for him to attain what he at time 0 considered to be a higher indifference level than the one actually maintained at that time. What one seeks is the least income required in the new price situation 1 in order that the consumer shall attain the original indifference level φ_0. This income divided by the original income gives what is often called the 'correct' index or the 'true' index.

If the quantities needed to attain φ_0 are designated $\bar{q}$, the following relation holds:

$$(1.17) \qquad I_{01}(\varphi_0) = \frac{\Sigma p_1 \bar{q}}{\Sigma p_0 q_0} < \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} = \text{Laspeyres' formula}$$

The inequality is a necessary consequence of the convexity of the indifference loci.

If the problem is simplified so as to refer to only two goods, a and b, it can be represented diagrammatically; see fig. 1.1.[5] The expenditure line AB is a tangent to the indifference curve. The point of tangency, P, represents the consumption in situation 0 (coordinates q_0^a and q_0^b). In the new price situation 1 the slope of the budget lines is that of $A'B'$. If the consumer has an income corresponding to the line $A'B'$, he will choose a different consumption, thereby attaining a higher indifference level. The 'true' index is obtained if the income corresponding to the budget line $A''B''$, parallel to $A'B'$ and tangential to the given indifference curve, is divided by the original income.

It should be noted that no reference has hitherto been made to the actual position of the consumer in situation 1. It must be assumed that in this new situation, in which the indifference map is assumed unaltered, the consumer finds himself on a different indifference level (higher or lower). A price index may also be defined from the indifference level (φ_1) in situation 1. One then obtains

$$(1.18) \qquad I_{01}(\varphi_1) = \frac{\Sigma p_1 q_1}{\Sigma p_0 \bar{q}} > \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} = \text{Paasche's formula}$$

[5] In this as in all other indifference diagrams in this book the indifference curves are full, whereas the budget lines are broken.

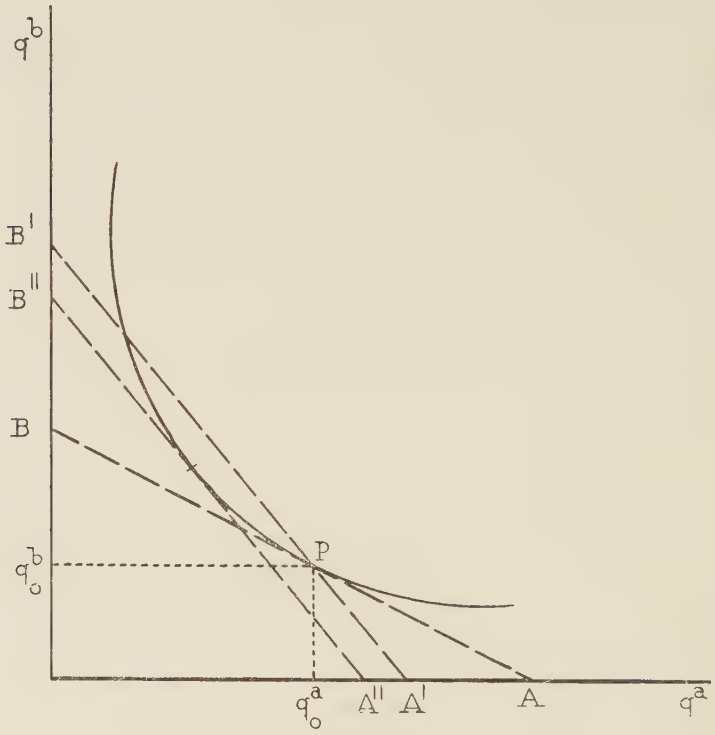

Fig. 1.1

The implication of this argument is that there are two indifference defined price indexes between 0 and 1; the one being based on the indifference level in 0, and the other on the level in 1. The only thing which one knows is that

(1.19) $$I_{01}(\varphi_0) < I^L_{01} \text{ and } I_{01}(\varphi_1) > I^P_{01}$$

Otherwise one may have

(1.20) $I_{01}(\varphi_0) \gtrless I_{01}(\varphi_1)$ and (1.21) $I^L_{01} \gtrless I^P_{01}$

If one knows that the indifference level is the same in the two situations one has $I_{01}(\varphi_0) = I_{01}(\varphi_1)$. But if this is the case no index problem exists; the price index will then equal the quotient between the incomes in the two situations.

The above-mentioned arguments are well-known to all students of the economic theory of index numbers; they have only been repeated here because it seems as if many computers of index numbers are not fully aware of this line of development.

Later research in this field has been devoted to an analysis of the inequalities given above and to different attempts to arrive at conditions under which it may be established that the price index lies between two limits. This literature need not be referred to here. Neither is there any reason to enter into the discussion in the literature upon the possibility of obtaining an exact solution of the index problem, as stated above, based on the assumption that the so-called Engel curves are known.[6]

Real wage comparisons

The indifference diagram gives a perfectly exact answer to the question: what income is required in price situation 1 in order that the consumer shall remain on the same indifference level as the one actually maintained in situation 0? In many cases the index thus defined is the suitable one. However, price indexes are also frequently used for the comparison of real wages or real incomes. The actual income change—from $\Sigma p_0 q_0$ to $\Sigma p_1 q_1$—is then compared with the index change and the quotient between these indexes is called the real income increase. It is immediately seen that a completely new element has been introduced when comparisons of this kind are undertaken.

If prices are unchanged and the income has changed, the consumer has the possibility of choosing any position on the new budget line parallel to the old one. The position actually chosen will in general not be situated on the straight line connecting the origin with $\Sigma p_0 q_0$, i.e. the quantities of the consumed goods will not have increased or decreased proportionately. The different positions on the new budget line represent different indifference levels, but they all imply the same income increase (from $\Sigma p_0 q_0$ to $\Sigma p_0 q_1$).

Real income change must be synonymous with volume or quantity

[6] See, for instance, *Wald* (1939).

increase. As pointed out by Allen, the volume index implies a numerical comparison of different indifference levels:

"The difficulty is... that the ordinal nature of 'utility' implies that the volume change is ordinal in nature. There is a multiplicity of measures of the change, though all measures rise or fall together.

We can limit the multiplicity of measures by adapting reasonable conventions. We can agree to use money values, i.e. to value the quantities purchased at certain prices. Fixing one set of prices (p), we define an expenditure line across the preference map joining the points (q) where a price plane with slopes (p) is tangential to the indifference surfaces. We can then measure the volume of consumption by taking indifference levels as ordered along this line and using distances between tangent planes (measured in the direction of the money or *numéraire* axis). A perfectly definite index of volume change is thus defined. But all this relates to the choice of prices (p). There is a measure of the volume index $Q_{01}(p)$ for each price set (p) selected."[7]

Allen subsequently gives the relations

$$Q_{01}(p_0) < \frac{\Sigma p_0 q_1}{\Sigma p_0 q_0} \qquad \text{(Laspeyres Form)} \tag{1.22}$$

and

$$Q_{01}(p_1) > \frac{\Sigma p_1 q_1}{\Sigma p_1 q_0} \qquad \text{(Paasche Form)} \tag{1.23}$$

which correspond to the relations (1.19).

In fig. 1.2 the budget line B represents an income which is as much higher than A as the income D is higher than C. If A is the original price situation and O the position of the consumer, the income corresponding to C gives the price index in the new price situation, as referred to the original indifference level. But R, the point of optimum satisfaction on the budget line D—D is tangential to an indifference curve at R—need not be on the same indifference curve as P, the point of optimum satisfaction for the budget line B.[8]

However, before continuing this line of thought, we shall describe the Divisia approach to the index problem.

[7] *Allen* (1949), p. 199. — Allen's notation has been slightly altered in order to conform with the notation used here.

[8] It is possible to state what form the indifference map shall have, if this shall be the case; see *Ville* (1946).

Fig. 1.2

Divisia's integral index

Divisia's contribution dates back as far as 1925[9], and became more widely known in 1936[10]. However, only a few of the later contributors to the theory of index numbers have been working along the same lines, one of the exceptions being Törnqvist who developed, independently of Divisia, a theory which is essentially identical though further pursued.[11]

Divisia gives no reference to the indifference theory. He aims at

[9] *Divisia* (1925).
[10] Through *Frisch* (1936).
[11] *Törnqvist* (1937).

finding the exact solution for an index showing "la valeur de la monnaie."[1] He holds that for society as a whole one must have

$$I_{01} \cdot Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \tag{1.24}$$

i.e. the product of the price index and the volume index must equal the value ratio (cf. the criterion 1.13).

After logarithmic differentiation one obtains

$$\frac{dI}{I} + \frac{dQ}{Q} = \frac{\Sigma q dp}{\Sigma pq} + \frac{\Sigma p dq}{\Sigma pq} \tag{1.25}$$

The price and the volume indexes may now be simultaneously defined, with the help of (1.25), by putting

$$\frac{dI}{I} = \frac{\Sigma q dp}{\Sigma qp} \quad \text{and} \quad \frac{dQ}{Q} = \frac{\Sigma p dq}{\Sigma pq} \tag{1.26, 1.27}$$

The price and the volume indexes thus take the form of differential equations, having curvilinear integrals as solutions.[2]

If we consider the price index, it will be noted that formula (1.26) may be obtained directly from formula (1.6), if it is assumed that the difference between the Laspeyres and the Paasche indexes is of the second order, i.e. during the infinitesimal time interval the q's are assumed constant[3]. One has then

$$\frac{I + dI}{I} = \frac{\Sigma (p + dp) q}{\Sigma pq} \tag{1.28}$$

which leads to formula (1.26). The integral which is the solution of formula (1.26) will be approximated by a chain index, the approximation being better, the shorter the links.

It remains to be considered which index formula shall be used for every link. With certain simplifying assumptions concerning the variation of the q's, it is possible to integrate formula (1.26) explicitly. In this way practically all index formulas may be obtained. Thus, if one assumes that the quantities q are constant during each link, the

[1] *Divisia* (1925), p. 842.
[2] See *Divisia* (1925), pp. 999–1004, and *Törnqvist* (1937), p. 80.
[3] See *Wold* (1952).

general arithmetic formula (1.6) is obtained. If one instead assumes that the weights pq are constant, the general geometric formula (1.3) is obtained.[4]

Geographical comparisons

The purport of the theoretical arguments will be clearer, if we again, and this time more thoroughly, consider the aim of the index comparison. Let us begin with geographical comparisons.

When two places, A and B, are compared, one may, as pointed out above, base the comparison on either A or B. The index problem may now be worded in the following way: if a consumer with a given income is chosen from A, what is the income required in order that this consumer in (price) situation B shall remain on the same indifference level as the one actually maintained at A?

It should be noticed that this statement of the problem strictly adheres to the conditions, i.e. the indifference maps and the indifference level, at place A. If, instead, place B is taken as the basis of the comparison, the problem is a completely different one, and the two results need not be the reciprocals of each other. It is not inconsistent if one comes to the conclusion that for an average person living in place A, place B is more expensive than place A, whereas at the same time place A is found to be more expensive than place B for an average person living in place B.[5]

If the A-consumer is actually removed to place B, he will behave in a different fashion from before. His actual consumption in place B will, however, not only depend upon the new relative prices, but also on the amount of money which he gets, i.e. his new indifference level, and on the new influences to which he may be subject in the new environment, i.e. the indifference maps have been revised. As these last-mentioned factors may be assumed to dominate, the actual behaviour of the consumer after a removal is of a minor interest for the index problem at hand.

If relative prices are approximately the same at place B as at place A and outer conditions (climate etc.) are equivalent, it seems

[4] See *Roy* (1927), pp. 1264–1277; cf. *Törnqvist* (1927), p. 81.

[5] Cf. *Principer* (1951), ch. 2, and *Burgson* (1949), pp. 253–255. Cf. *Staehle* (1949).

legitimate to use Laspeyres' formula; i.e. to compute how much it would cost to live in place B in exactly the same way as in place A. The more different the relations between the prices at the two places, the more evident it will be that the Laspeyres comparison will give too high a result. In order to allow for this bias one may make substitutions, e.g. substitute wine for beer. One may also make quantity adjustments, thus we may take into consideration the difference in environment, e.g. different requirements of fuel for different climates, more petrol in a large city than in a small etc.[6] It should be noticed, however, that such substitutions or adjustments should be based on the conditions at A, i.e. the valuation of the items for consumers living at that place. It is consequently not recommendable to make use of the quantities q_1 in this connection.

Time comparisons

When time comparisons are considered, the index problem *may* be stated in a similar way, i.e. what one wants to know is the income required at a—generally, but not necessarily—later point in time in order that the consumer shall remain at the given indifference level. If the points in time are remote and relative prices quite different, one will again find that the bias introduced through the use of Laspeyres' formula must be considerable and that consequently some kind of substitution or adjustment should be applied in order to obtain a better approximation for $I_{01}(\varphi_0)$. However, if the problem is stated in this way, one may not regard the actual income increase divided by the price index as a *measure* of the real income increase (cf. fig. 1.2).

When studying the temporal development of prices and consumption habits one finds that the changes may under certain circumstances, e.g. the introduction of a rationing scheme, or a very rapid inflationary process, be sudden. As a rule, however, changes in prices, incomes, and consumption habits within a moderate time interval are so small that the process can be described as continuous.

The Divisia integral index and its approximation, the chain index, would from these considerations seem to be applicable to time com-

[6] Cf. *Woodbury* (1940).

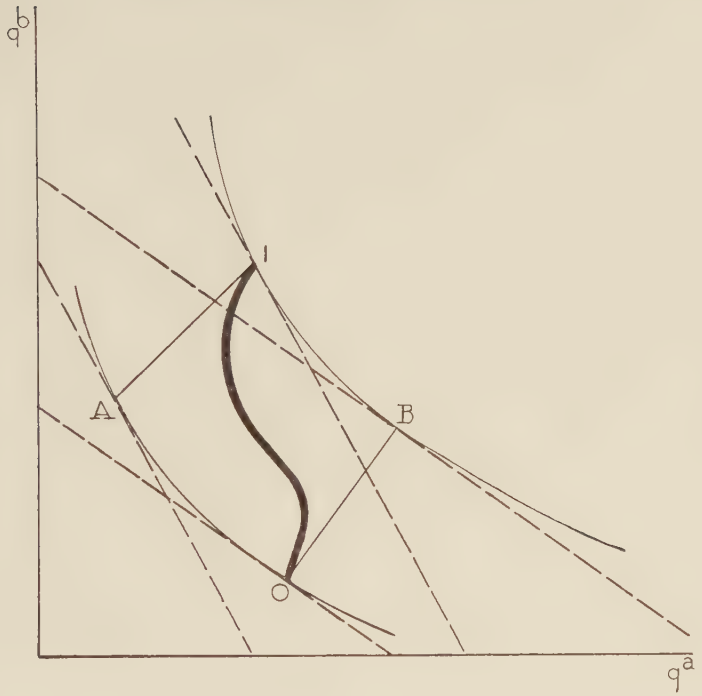

Fig. 1.3

parisons. Divisia strongly advocates this point of view when dealing with historical comparisons. Thus he says:

"Supposons que nous connaissions toutes les quantités de marchandises et de services vendues en 1800 ainsi que leurs prix et de même en 1900. Nous voulons calculer la valeur de la monnaie en 1900 par rapport à sa valeur en 1800; c'est bien là l'objet des index. Eh bien! les données dont nous disposons ne permettent par de résoudre le problème.

En effet, la valeur de l'intégrale curviligne ne dépend pas seulement de la valeur des variables en 1800 et en 1900, elle dépend encore *de toutes les valeurs intermédiaires.*"[7]

The Divisia approach may evidently well be applied to an indifference map.[8] The conclusion will be that the indifference

[7] *Divisia* (1925), p. 1005.
[8] *Roy* (1942).

defined 'true' index is only a special case of the more general integral solution. This is seen if fig. 1.3 is considered.

Here o is the actual position of the consumer at the point in time o. This point, as well as the actual path which the consumer has traced in the time interval from o to 1 may, at least in principle, be ascertained by observation, and has been sketched in the diagram. The Divisia price index will be the line integral of prices along this path. In a similar way the Divisia volume or real income index will equal the line integral of quantities along the same path.

The indifference defined price index, based on the indifference level in situation o, is now seen to equal the line integral along the indifference curve through o, i.e. one *assumes* that the consumer has moved from o to *A*. But in situation 1 the consumer was at 1, and this assumption must consequently imply that the time interval from o to 1 can be divided in two intervals, the one from o to *A*, and the other from *A* to 1. During the first interval the price change occurs and the consumer is just compensated. During the second interval, when prices are unchanged, the consumer receives an income increase which brings him from *A* to 1, i.e. the real income change is measured in the new prices at 1.—In a similar way it may also be assumed that the consumer goes from o to 1 via *B*, i.e. that the volume change takes place first and is measured in the o-prices. Afterwards the price change occurs and is measured at the new indifference level 1.

It is thus seen that the indifference defined index, whether it be based on the indifference level at o or at 1, is founded upon a specific condition concerning the way in which price and quantity changes have taken place. In practice one cannot, of course, assume that these conditions are fulfilled.

The conclusion is that the indifference defined index is based on a condition which is just as special as the condition underlying the Laspeyres or the Paasche indexes. The Laspeyres comparison is obviously founded on the assumption that the consumer all the time remains at the point o, and the Paasche comparison that he remains at point 1. In a similar way Edgeworth's formula implies the assumption that the consumer remains at a point which is mid-way between o and 1.

The Divisia integral index as a matter of fact corresponds much more nearly to the problem which the index should solve. There is, however, one difficulty, viz. that the integral index need not return to the starting-point, if all prices have returned to their original values. As shown by Ville this will be the case if and only if the utility functions are homogeneous functions, which need not hold.[9]

This difficulty is especially obvious if seasonal fluctuations are considered. It is unnatural to accept an index which may after a year give a result different from 1, if the prices have returned to their initial values. Divisia also says:

"Il semble que l'on ne ferait que compliquer inutilement l'étude du fonctionnement de la loi circulatoire en suivant ses facteurs à travers ces oscillations et que l'on doive ainsi pratiquement adopter pour unité de temps au moins l'année."[1]

Yearly links should then be used. If this is done, one will hardly find any example of a case when prices (relative prices) have returned to the starting-point. It is possible, however, to imagine a country where relative prices, after a period of disturbance, such as a war, have returned to approximately the original values, and where the integral index gives a result which is not in agreement with the price ratio for the different items. One may also imagine a case where a country is split into two, and where after a period, during which the economic development followed very different courses in the two parts, these are again united. The integral indexes for the two parts may then show a different result.

In spite of these difficulties it does not seem unreasonable to accept the integral index as giving the answer to the question which is in general the relevant one when time comparisons are considered. This does not exclude the use of direct 'point-to-point' comparisons, based on the indifference level of either situation. It should be noted, however, that if the integral index and the direct comparison based on the indifference level of situation o should differ, the two results may not be checked against each other. As the two indexes answer different questions, they may well give different answers.

[9] *Ville* (1946), p. 37.
[1] *Divisia* (1925), p. 1007.

A few more words may be added concerning geographical comparisons. Here the integral index seems to have no application. A continuous translation from one place to another is impossible.[2]

But even if the integral index is not applicable to geographical comparisons in the sense that it does not describe any path really followed by the consumer, this solution nevertheless seems to be applicable when a comparison between both prices and quantities in two countries shall be undertaken. Under the assumption that the indifference maps are the same in the two places, it is reasonable to found the comparison on the price and quantity indexes which are obtained if the consumer takes the shortest path between the two points 0 and 1. In this way the expenditure difference between the two places can be split up in a price and a volume component.

The problem of limits

In the earlier literature (Fisher etc.) it was generally assumed that there is only one 'correct' price index between two situations 0 and 1, and that it must always be included between the limits given by the Laspeyres and the Paasche formulas. In the more recent literature it has been emphasized that this need not be true if the indifference defined index referring to the indifference level of either situation 0 and 1 is contemplated. However, if the index is defined in the Divisia way, it may be possible to state conditions which will guarantee that this index is included between certain limits. Although no such attempt will be made here, a few hints in this direction will be given.

We assume that $I(\varphi_0)$ and $I(\varphi_1)$ are different.[3] I^D (= the Divisia price index) may be regarded as a weighted average of the price indexes for the indifference levels along the path which the consumer has followed from 0 to 1. It is a reasonable assumption that for all values of φ between φ_0 and φ_1 the price index has a value between $I(\varphi_0)$ and $I(\varphi_1)$. If then, in the interval from 0 to 1, the consumer has

[2] Within a country such a translation may be given a meaning, although of course in several different ways, giving different results. But when the comparison includes different countries, the transition from the one currency to the other must always be discontinuous.

[3] The subscripts 01 have been omitted here.

not overstepped the limits given by φ_0 and φ_1—a war period between 0 and 1 is thus excluded—I^D must have a value between $I(\varphi_0)$ and $I(\varphi_1)$.

However, this does not imply that I^D is included between I^L and I^P, as may be seen from Konüs' inequalities, according to which $I(\varphi_0)$, $I(\varphi_1)$, I^L and I^P can be ordered in six different ways and still fulfill (1.19)[4]:

1) $I^P < I(\varphi_1) < I(\varphi_0) < I^L$
2) $I^P < I(\varphi_0) < I(\varphi_1) < I^L$
3) $I(\varphi_0) < I^P < I(\varphi_1) < I^L$
4) $I^P < I(\varphi_0) < I^L < I(\varphi_1)$
5) $I(\varphi_0) < I^P < I^L < I(\varphi_1)$
6) $I(\varphi_0) < I^L < I^P < I(\varphi_1)$

It is only in cases 1) and 2) that the fact that I^D is included between $I(\varphi_0)$ and $I(\varphi_1)$ necessarily implies that I^D must be included between I^L and I^P.

We further note that the following relation must be fulfilled:

$$(1.29)\quad I^L \cdot Q^P = I^P \cdot Q^L = I(\varphi_0) \cdot I(p_1) = I(\varphi_1) \cdot Q(p_0) = \\ = I^D \cdot Q^D = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

The six cases given above for the price indexes thus correspond to the following six cases for the quantity indexes:

1) $Q^P < Q(p_1) < Q(p_0) < Q^L$
2) $Q^P < Q(p_0) < Q(p_1) < Q^L$
3) $Q^P < Q(p_0) < Q^L < Q(p_1)$
4) $Q(p_0) < Q^P < Q(p_1) < Q^L$
5) $Q(p_0) < Q^P < Q^L < Q(p_1)$
6) $Q(p_0) < Q^L < Q^P < Q(p_1)$

The inequalities (1.22) and (1.23) are here satisfied. Here again, if Q^D is included between $Q(p_0)$ and $Q(p_1)$, it will only in cases 1) and 2) necessarily follow that I^D is also included between Q^L and Q^P.

It is consequently not possible to state that I^D is included between I^L and I^P, without the introduction of additional assumptions. In practice it seems nevertheless to be sound to assume that I^D may be approximated by an index figure between I^L and I^P, e.g. the index as given by formulas such as Edgeworth's, (1.11), or Fisher's ideal formula.

[4] *Konüs* (1939), pp. 20—21.

Chapter 2.—Price Index Problems in Practice

The vigorous discussion of the theory of price indexes has only had a comparatively small influence on the construction of the regular index series. On the other hand, many of the problems which the index series computer encounters have not been considered to be of interest for the theoretical discussion. The following statement by Frisch in his survey article on problems of index numbers may be quoted as illustration: "Only the fundamental theoretical problems will be considered. We shall not take up practical questions connected with the reliability or fullness of the data nor discuss questions arising out of the practical necessity of using a limited number of *representative* articles."[1]

Actual index series

Regular price index series are computed and published in most countries. Two main types of such indexes may be distinguished, namely wholesale price indexes and retail price indexes.

The first regular price index series were computed during the latter part of the nineteenth century. These earlier series covered wholesale prices only. Later on came series covering prices for food and other sections of the retail market. Regular price index series, intended to cover all kinds of commodities in the retail market, were, however, not computed until during World War I[2]. Round about 75 such indexes are now computed and regularly published.[3]

[1] *Frisch* (1936), p. 2.

[2] For a brief history, see *Ulmer* (1949), pp. 26–27. See also *Fisher* (1922), pp. 458–460, and *Mitchell* (1921), pp. 7–8.

[3] Basic facts about the construction of these indexes are given in the *International Labour Review*, June–November 1948, and in *Supplement* (1950). See also *Hofsten* (1949), p. 143.

Most of these series are called cost-of-living indexes. Originally this concept was limited to computations referring only to the working classes. In some cases such indexes were originally not even pure price indexes, but also showed increases and decreases in expenditure. Even when in principle price indexes they often referred only to part of the working class consumption, viz. that part considered as necessary for the attainment of a certain minimum living standard.[4]

Modern development, and especially that stimulated by Keynes, has resulted in a demand for retail price indexes of a more general character, aiming at comparisons of the purchasing power of money at different times.[5] In some cases this demand has created quite new index series parallel to the cost-of-living indexes.[6] In most countries the cost-of-living index series are now of a fairly general character. Even if most of these indexes still refer only to a limited section of the population, they are—at least in principle—pure price indexes, meant to cover all kinds of expenditure for the group in question.

Index nomenclature

It is not possible to distinguish between different types of retail price indexes by the name only. Irrespective of its name, such an index may refer to a special section of the population or be of a more general character. An index may also have a specific meaning in other respects. Certain kinds of expenditure may for instance more or less consciously have been excluded from the computations. That is the case with direct taxes in most countries.[7]

[4] It seems as if Keynes thought that this latter restriction was valid for all indexes, which could be termed cost-of-living indexes; see *Keynes* (1930), p. 64. This is, however, an exaggeration.

[5] *Keynes* (1930), p. 57.

[6] In Sweden, besides the old cost-of-living index, a more general consumers' price index has been computed since 1931 (*socialstyrelsens allmänna konsumtionsprisindex*, previously called *riksbankens konsumtionsprisindex*). For literature see *Lindahl* (1933), *Lindahl and Lundberg* (1938), and *Öberg* (1949). A similar index was computed in Finland from 1931 to 1945; see *Törnqvist* (1936).

[7] The only countries, in which only retail price indexes including direct (income) taxes are published, seem to be Curaçao, Czechoslovakia, Denmark, Fiji, Lebanon, Netherlands, and Union of South Africa. Source: see note 3. In Sweden—and from 1951 also in Finland—the cost-of-living index is published in two series, the one with and the other without direct taxes. The Swedish consumer's price index does not include direct taxes.

The name cost-of-living index is still used for most series, but in some countries it has been found that another name better corresponds to the character of the index. Thus, the US index is now called the *Consumers Price Index* (CPI), and the name of the corresponding British index is the (Interim) *Index of Retail Prices.*

The existing confusion with respect to the nomenclature is well shown in the hearings before a Subcommittee of the American Congress which took place in 1951. Some of the witnesses expressed the opinion that the term cost-of-living index should be used for an index covering all changes in the consumers' expenditure, i.e. including changes in the standard of living, whereas the term price index should be used for series intended to describe the effect of price changes only.[8] This view seems to have been accepted by the Subcommittee itself.[9] The views were also expressed that the two concepts should be considered as identical[1], and that a cost-of-living index is a price index with continually varying weights.[2] Finally, in a written statement to the Subcommittee, Ewan Clague, Commissioner of the Bureau of Labor Statistics, claimed that "in the cost-of-living index we would try to measure the changes in the cost of an equivalent market basket of goods and services, whereas in a price index we try to measure the same market basket."[3]

None of these terminological distictions is, however, generally accepted either in the United States or in other countries.

The question of the most appropriate name for price indexes of the kind treated here was one of the topics raised when the Sixth International Conference of Labour Statisticians, held at Montreal in 1947, discussed the price index problems. In one of the draft resolutions presented to the Conference by the International Labour Office a positive recommendation had been worded, but agreement could only be reached on what is contained in paragraph 15 of Resolution II: "In order to promote understanding of the nature and uses of indices of retail prices charged a particular group, the term 'cost-of-living index' should be replaced, in appropriate circum-

[8] *Hearings* (1951), p. 181 (Cowden).
[9] *Report* (1951), pp. 1—2.
[1] *Hearings* (1951), p. 102 (Mudgett).
[2] *Hearings* (1951), p. 172 (Cox).
[3] *Hearings* (1951), p. 391.

stances, by the term 'price-of-living index', 'cost-of-living price index', or 'consumer price index'."[4]

From the different names proposed it is understood that the Conference wanted it to be quite clear that such indexes should be price indexes. This view is now generally accepted among experts, but not always among those making use of the figures.

The quoted resolution has in many other respects contributed towards a clearer understanding of the requirements which a retail price index should fulfil. The general aim is given in paragraph 1(1) of the above-mentioned Resolution II: "The object of cost-of-living index numbers should be to measure the changes over time in retail prices of a given standard of living."[5]

Similar definitions can be found in the descriptions of different national indexes. It can thus be mentioned that the official Swedish committee, appointed in 1942 in order to revise the cost-of-living index for Sweden, stated in its report that the index calculations should "refer to the same standard of living at the time of calculation as at the time with which the price comparison primarily takes place" ("avse samma levnadsstandard vid beräkningstillfället som vid den tidpunkt, med vilken prisjämförelsen närmast sker").[6] A corresponding American committee in its report stated that "The purpose of the BLS 'cost-of-living index' is to measure *changes* in the cost of a *constant* (observed and measured) standard of living".[7]

Misunderstandings concerning the general aim of a retail price index are, however, very common. As an example of a kind of clarification, which must often be made, the following extract from a popular description of the Canadian index is given: "The important point to remember is that the index measures changing *costs* of living and not changing *standards* of living. It is an index of price movements and not of changes in total family expenditures. This may be the cause of some of the misunderstanding, for Canadians as a whole

[4] *Conference* (1948), p. 62. The following references to this resolution come from the same source. Cf. *Cost-of-living* (1947), where the proposed resolutions are to be found.

[5] This definition is seen to be well in accordance with the corresponding definitions given in the theoretical literature based upon the Konüs approach; cf. chapter 1.

[6] *Betänkande* (1943), p. 7.

[7] *Report* (1945), p. 6.

have increased their standard of living during the past ten years and it is easy to confuse the greater cost of a higher standard with an increase in the prices of the goods and services in a fixed family budget."[8]

A list of problems

The laying down of the general principle that the index shall be a price index and not an expenditure index is, however, only the first step in the establishment of a complete series of index numbers. Many other problems must be solved, the most important of which have been collected in the following list.

1) It must be made clear what should be meant by 'a given standard of living,' i.e. how it should be measured and to what time and to what section of the population it should refer.

2) The procedure of comparing two times which differ not only in prices but also in standard of living and consumption habits should be clarified.

3) The index should be computed at regular intervals and every new index figure should be a measure of the price change not only with reference to the base of the series but also with reference to all previous index figures and especially to the one immediately previous to it.

4) The goods[9] on the market and the conditions of living vary during different seasons of the year, and it is consequently difficult to form comparisons between different seasons.

5) Even if comparisons are restricted to whole years or to certain seasons of the year, the goods on the market may vary considerably from time to time.

6) The introduction or improvement of social security schemes or public utilities may make certain expenditure unnecessary or in some other way give the consumer benefits, which are hard to evaluate but which should be taken into account when price index comparisons are undertaken. The same is true if the value of such benefits is reduced.

[8] *Cost-of-living* (1948).—Similar quotations could be cited from *Hearings* (1951) and *Report* (1951).

[9] The word 'goods' is here as elsewhere used in the economic sense, i.e. it includes services, such as hair-cuts, bus rides, cinema tickets, etc.

7) With respect to taxes similar problems arise.

8) During periods of rationing or when consumption is otherwise restricted, special problems for price index comparisons arise.

9) The index should refer to the price level at a certain point in time or during a short period, but consumption includes durables which last long.

10) Prices cannot be obtained in practice for all goods on the retail market.

11) The price quotations may be subject to errors of different kinds.

12) The consumption habits are in general not known with sufficient accuracy.

Of these problems the first three have—as can be seen in chapter 1—been treated thoroughly and systematically in the literature. The remaining problems have also been mentioned in the literature, though most of them only incidentally. This is especially true of the problems 5) and 10), which—although mentioned in the literature on several occasions—do not seem to have been treated systematically before. The remaining part of this book will be mainly devoted to those two problems together with necessary reconsideration of the fundamental problems 1), 2), and 3). Before entering into a discussion of the quality problem some more space will, however, be devoted to the Montreal resolutions.

The Montreal recommendations

The quoted paragraph 1(1) of the Montreal Resolution II is accompanied by paragraph 1(2), which is worded: "This in practice means that the index should measure the changes in retail prices of a given consumption pattern." Next it is stated in paragraph 2(1) that "The pattern of consumption to which any particular index relates should be clearly defined." It is thus made clear that 'the given standard of living' should be defined through information on consumption habits. It can be said that the general arithmetic formula (1.6) is recommended.

The question of the epoch to which the q-values should refer is treated in paragraph 3, which states that

"(1) The purposes which may be served by such indices in the post-war period are the measuring of changes over time in retail prices currently charged consumers for—

(a) the pre-war consumption level of a given economic group in specified types of communities;

(b) a post-war consumption level of a given economic group in specified types of communities; or

(c) a consumption level of a given standard taking account of price induced substitutions and any other substitutions imposed by circumstances, with as close regard to the maintenance of the same standard of adequacy as practicable.

(2) In cases where a post-war consumption level is adopted, the index may—

(a) measure the variations in the cost of maintaining unchanged the pattern of consumption in a specific year or period; or

(b) measure price changes using the current pattern of consumption as a weighting diagram adjusted as necessary to conform to changes in the consumption pattern."

In more technical language this recommendation can be said to imply that the following alternatives are considered permissible:

1) Laspeyres' formula.

2) Paasche's formula, though not for comparisons including the pre-war period.

3) Certain modifications of these formulas.

In practice Laspeyres' formula is used for almost all retail price indexes.[1]

According to the discussion in chapter 1, it is legitimate to use Laspeyres' formula, if the price development for the different items has not been too heterogenous. In practice this means that the q-values may under normal conditions be kept constant for a number of years. However, sooner or later one comes to a point where it becomes necessary to revise the weights. This is especially obvious, if it is realized that, in a series of index numbers, not only are all figures compared with the base figure, but also the different figures in the series are compared with one another. If Laspeyres' formula

[1] See previously quoted reference in *International Labour Review*. Sweden seems to be the only country where—at least in principle—a different formula is used; cf. the introduction to chapter 4.

is used, the comparison between the indexes for times $t-1$ and t will be based on the expression

$$\frac{I_{0t}}{I_{0,t-1}} = \frac{\Sigma p_t q_0}{\Sigma p_{t-1} q_0}$$

This price comparison will thus be based on quantities which may be completely out of date.

If Paasche's formula is used, the quotient between the indexes obtained for times $t-1$ and t will be the expression

$$\frac{I_{0t}}{I_{0,t-1}} = \frac{\Sigma p_t q_t}{\Sigma p_0 q_t} \cdot \frac{\Sigma p_0 q_{t-1}}{\Sigma p_{t-1} q_{t-1}}$$

Neither will this expression assume values which can be accepted as a measure of the development between $t-1$ and t.

That the weights must be revised at moderate intervals was realized in the Montreal resolution. Thus paragraph 3(3) states: "Where a decision is taken to measure the variations in the cost of maintaining unchanged the pattern of consumption in a specific year, the pattern of consumption should be examined, and the weighting diagram adjusted, if necessary, at intervals of not more than ten years to correspond with the changes in the consumption pattern."

It is sometimes argued that an entirely new series should be begun when the weighting diagram is adjusted. This procedure was, as a matter of fact, recommended by the Second International Conference of Labour Statisticians, which stated: "Changes in the cost of living in a given period should be calculated by using weights which remain unchanged throughout the period. When changes in consumption render the weights inappropriate a new series of index numbers based on new weights should be calculated."[2]

As an example may be mentioned, that when the weighting diagram of the U. S. index was revised in 1940 a new series in which 1935–39 = 100 was started. At this occasion not only was the old series in which 1923–25 = 100 discontinued, but also the new series was computed back as far as 1913.[3]

[2] *Conference* (1925), p. 69; cf. *International* (1943), p. 98.
[3] *Hearings* (1951), p. 8.

Nowadays, however, it has become usual not to alter the 'base of publication' after a revision of the weights, i.e. the two series are spliced together. This procedure is as a matter of fact recommended in the Montreal resolution, where paragraph 4 is worded: "In countries where it is decided to introduce a new weighting diagram, consideration should be given to splicing the new indices to the old series dating back as far as possible."

Every index series, where a revision of the weights has been undertaken at least once, may be classified as a chain index. The term chain index is, however, usually restricted to series, where the weights are regularly revised at fairly short intervals.

According to the final result obtained in chapter 1 there is some evidence in favour of a method which gives a result between the Laspeyres and the Paasche indexes. As a rule the figure obtained in this way will be lower than the one obtained through the use of Laspeyres' formula. Instances where Paache's formula gives a higher result than Laspeyres' formula are, however, by no means exceptional. As an example the following figures for butter and margarine in Sweden are given.[4]

	1950 (July—Nov.)		1951 (Febr.—April)		
	Price p_0 (öre/kg)	Quantity q_0 (million kg per month)	Price p_1 (öre/kg)	Quantity q_1 (million kg per month)	$\frac{p_1}{p_0} \cdot 100$
Butter	560	8.1	645	6.3	115
Margarine	260	4.7	345	5.9	133

In spite of the price increase's having been larger for margarine than for butter, the consumers substituted margarine for butter. This tendency is not inconsistent with the indifference theory as developed in the previous chapter. The interpretation will be that the reduced indifference level made the transition from butter to margarine advantageous.[5] The figures give $I^L = 118.9$ and $I^P = 120.5$. Edgeworth's formula as well as Fisher's ideal formula here give 119.7.

[4] The forthcoming price changes which took place January 1st became known in December, and as a consequence consumption was abnormal in December and January. For this reason these two months have been excluded in the example.

[5] A similar case is treated in *Bowley* (1940—41).

In practice the q-values are often known only with very little accuracy. When the comparison refers to a short interval, the uncertainty in the q_0-values may often be much greater than the hypothetical difference between q_0 and q_1. To undertake in such cases a yearly revision of the weights is hardly reasonable. But if after a certain number of years it is found that the q-values could and should be revised, the best method seems to be to splice the new series—based on new Laspeyres weights—not to the last figure in the old series but to a revised figure at the end of the old series. This revised figure may be founded on Edgeworth's formula, Fisher's ideal formula etc.

The two Swedish index series are both chain indexes with yearly links. In both series the indexes for the separate links are founded on formula (1.11). This procedure is seen to be well in accordance with the recommendation given above.

Chapter 3.—The Quality Problem in Outline

The analysis set forth in the preceding chapters presupposes that the same goods are to be found for sale on the retail market at both the points in time (or places) which are compared, and that price quotations are obtained for all these goods. In practice neither of these conditions is satisfied.

In the subsequent treatment of these two problems, which have, as a matter of fact, an intimate connection, the theoretical ideas of chapter 1 will not be considered initially. The index is defined in the general arithmetic way, i.e. a budget is assumed given and the object is to compare the cost for that budget in different price situations.

The items in the budget

In a budget quantity or outlay figures will be given for items such as cheese, woollen material, wireless sets, postage etc. In that way every quality of every article can be said to be included in the budget.

Certain budget items, such as butter, milk, coke, etc. represent rather uniform articles which are only present on the market in one or a few varieties, whose prices are all obtainable. But many other budget items, such as medicine, toys, sporting articles, embrace a great number of articles. A budget cannot be subdivided so far as to show separately all the articles included in such groups. This is still more evident, if budget items, such as woollen material, razor blades, cigarettes, or wireless sets, are considered. Here the number of accurately defined articles included in one budget item is extremely large and it is impracticable to distinguish between the constituent items. Even during the war woollen material could be obtained in several hundred varieties in Sweden; now the number is much

greater.[1] Considering razor blades, between 200 and 300 different kinds are for sale in Sweden, at least to judge from the wrappings. A considerable number of these blades may in reality be identical, but what matters here is that they are sold as if they were all different.

A price quotation must refer to a strictly defined article. There is nothing like a price for sporting articles, for woolen material etc., or, at least, the index comparison cannot be founded upon prices for articles which are so vaguely defined.

It is thus seen that the number of price quotations which may be included in any one actual index comparison must be quite small compared with the total number of articles on the market.[2]

The sampling problem

It is often assumed that the greater the number of price quotations, the more reliable the index. According to Divisia the International Statistical Institute has adopted a resolution, in which it is stated: "Le nombre des articles devrait être aussi grand que possible, sous la réserve que l'introduction de nouveaux articles ne diminue pas la précision moyenne de l'indice des prix."[3]

If the index comparison really aims at measuring the total cost-of-living, it is, of course, not sufficient that the number of price quotations should be large. All important items in a suitably subdivided budget must then be represented by one or several price quotations for accurately described articles.

On account of the practical difficulties of deciding upon appropriate price quotation articles, one may be tempted to exclude a number of budget items from the index calculations altogether. As a matter of fact many actual index series have a very poor representa-

[1] Cf. a statement by N. Kärrlander. *Nationalekonomiska föreningens förhandlingar* 1942, p. 93.

[2] In the earlier quoted specification in the *International Labour Review* information on the number of price quotation items is given for 71 index series. Of these 29 are founded on less than 50 items. Only 7 indexes are founded on more than 200 items, among them the Swedish cost-of-living index (236 items). The greatest number occurs in the Dutch index (515 items), but while visiting the Hague in 1949 the author was informed that the intention was to reduce this number on account of the difficulties of working with so many items.

[3] *Divisia* (1925), p. 1126.

tion among goods belonging to the 'miscellaneous group'. Such an index may give a result which deviates considerably from an index comparison based on a more complete budget.

Let us now consider a specific budget item such as postage. This heading covers a considerable number of items (postage for an ordinary letter, for a post-card, for a letter abroad, for printed matter, for parcels etc.). For practical reasons it may not be possible, for a small budget item like postage, to include more than one, or possibly a few, of these items in the index calculations. The selection of one item, such as, for instance, "internal postage for a letter not weighing more than 20 grammes," would imply the assumption that the price development for other postage items will in the future be the same as for the selected item. If this is the case, the index will give the same result as if all postage items had been included. It may thus seem as if the important thing would be to select the items to be priced in such a way as to guarantee a good sample.[4] One finds in practice, however, that the selection of items in current price index series is based on common sense and not on proper sampling methods. A consideration of the remaining problems, which will be undertaken presently, will show that this is no serious drawback. Moreover, the use of sampling would be expensive, as it would require complete lists of commodities.

There is also another sampling problem involved here, viz. the selection of retail outlets where the prices shall be collected. In a large city only a few shops may be visited and the price of a single article may vary considerably from shop to shop or, at least, from district to district. Although the difference in price *change* may not be so important, the selection of retail outlets will have a certain influence on the index. To be satisfactory the price collection should be based upon an efficient sample of the retail outlets. The construction of such a sample cannot be too difficult.

That it is important to pay attention to the selection of the items which are to be priced, has been remarked at both the ILO conferences, which have treated price index problems. In the resolution, adopted by the Second conference, it is thus stated that "each group

[4] This point of view is stressed by *Mudgett* (1951). He does not, however, consider the remaining problems which will be discussed presently.

should include the most important items of consumption, and especially those articles the price changes of which may be taken as representative of changes in the prices of articles not included."[5] The corresponding resolution, adopted by the Sixth conference, contains the following paragraph 11:

"(1) In establishing the weighting diagram for a cost-of-living index for a particular group, individual items priced should be assigned weights corresponding to the consumption expenditures not only on the individual items themselves but also on other articles not priced, in accordance with the principle that the weights for items not priced may be added to the weights for those priced, when the price movements of the latter are representative of the price movements of the former.

(2) In this manner, all items purchased by the group covered can be represented in the index, although not all are priced."[6]

But, however important it may be to select the goods and the retail outlets in a suitable fashion, two still more vital problems remain.

The first problem is this. If the items which have been selected for time o are published—which is often necessary and desirable—the price development of these items may, as a consequence of publication, differ from that of the goods which are not priced. A tendency of this character is especially to be feared during periods of price control or when prices are in other ways regulated by the government. The authorities may be more complient concerning price increases for goods which are not among the items priced. This problem can only be solved by keeping the selected items secret—also from the authorities dealing with price control! If this is not feasible, the danger of a bias will be lessened if the number of selected items is fairly high.

As an example it may be mentioned that the fact that at that time only milk was included in the official Swedish index, and not cream, may have influenced the authorities in their decision to raise the price of cream from 2.20 to 2.40 kr per liter as from January 1st 1949 (Stockholm prices). The price of milk was unchanged, 0.36 kr per liter, and the index was consequently not affected. Cream has later

[5] *Conference* (1925), p. 69.
[6] *Conference* (1948), p. 62.

been introduced in the Swedish index and has a cost-weight which is approximately one fifth of the cost-weight for milk.—The wide possibilities of in this way influencing the index are clearly demonstrated by the tables at the end of chapter 4; it is seen that the price development for many related articles was quite different from 1946 to 1949.

The second problem is the quality problem. If a certain selection of items to be priced referring to time 0 has been undertaken, what is to be done when it is found that some of the items are no longer obtainable at time 1 or subsequent times?

The quality problem

The totality of goods on the market is subject to constant change. Such changes have been most apparent during periods of war, when many goods have disappeared from the market, and during after-war periods, when they have returned. But, also, during more normal periods it is characteristic of the present-day retail trade that the totality of goods perpetually changes by the introduction of new goods and the disappearance of older ones. Such changes are least marked for food and fuel. However, for clothing, furniture, household utensils etc. one finds that a process is always going on, by which new varieties (models, brands etc.) are added to the market, whereas simultaneously some, at least, of the older products disappear. One notices that for quite a number of goods price changes for a strictly defined article hardly occur. The price is changed when a new model or make is introduced on the market.

This forms a problem for the computation of price indexes, a problem which has not been considered properly in the literature. It is not even clear how problems of this kind are handled in most of the current price index series. As will be demonstrated in chapter 4 the method of treating the quality problem has a strong effect on the numerical results.

It should be noted that articles for which the problem of quality changes is of importance play a much greater part now than they did some decades ago. Whereas consumption was formerly dominated by simple food items, it is now in well industrialized countries much

more varied, and the clothing and miscellaneous groups play a much greater part than hitherto.

References in the literature

One of the few authors, who has paid attention to the quality problem, is Knibbs, who already in 1912 wrote the following:

"The advances in technology have led in many cases to marked improvement in the quality of the manufactured articles. Since, however, the use of the article, thus improved in quality, may be continuous, and the change in quality may proceed by imperceptible changes, a feature not uncommon with regard to textiles, for example, it is not always possible to take so exact an account of differences of quality as has been indicated as necessary."[7]

Knibbs recommended that goods, which altered their quality, should successively be replaced in the index, i.e. he recommended what will here be called the splicing method.

Twelve years later Knibbs again treated the quality problem and then said:

"For example, in textiles a tweed at the second date may have the same general appearance and weight and replace the use of that at the first date, but the number of threads in the warp and woof may be wholly different and this may, of course, have affected the price. Such a case of pseudo-identity may be treated in one of two ways according to the purpose of the investigation. For one economic purpose it may be regarded as not a case of identity of regimen, since the difference in price is due to difference in quality, and would vitiate the deduction of a change in the purchasing-efficiency of money. For another economic purpose, however, we may ignore this difference, and treat it as if it were identical, and deduce the purchasing-efficiency of money regarded as under the compulsions of social habit, which, be it said, are continually replacing the usage-commodities of any date by others of different quality, or even of different character."[8]

As will be seen later on this statement is well in accordance with the conclusions arrived at in this book.

Another author, which should be quoted here, is Divisia. After having stated the problem, he says:

[7] *Knibbs* (1912), appendix, p. xliii.
[8] *Knibbs* (1924), p. 49.

"Reconnaissons d'ailleurs que même avec de telles précautions, il n'en subsistera pas moins un certain nombre d'articles d'identité douteuse. Mais l'on ne doit pas rencontrer de ce fait d'obstacle sérieux à la construction de l'indice *si l'on a soin de faire les comparaisons de l'indice monétaire à des intervalles très rapprochés, ainsi que le recommande expressément la définition même de l'indice.* Comme les changements de nature de certains articles, changements de rendement de certaines mains-d'œuvre, etc., ne se produisent pas tous en même temps, les cas douteux resteront toujours fort peu nombreux, dans un petit intervalle de temps, et l'on pourra les éliminer dans cet intervalle sans altérer sensiblement la valeur de l'indice."[9]

As will be demonstrated later the problem is not so easy to solve as Divisia thought.

The quality problem has been treated, though only superficially, at both the International conferences of labour statisticians where index problems have been on the agenda (the Second and the Sixth). The Second conference in 1925 adopted a resolution, in which paragraph 4 is worded in the following way:

"Care should be taken to ensure that the quality of each item for which prices are obtained at different dates and the methods of collection and calculation remain unchanged during the period covered."[1]

The prevailing ignorance of the difficulties of the problem is revealed by the fact that this paragraph did not cause any discussion to speak of and that there was only a slight amendment to the draft of the paragraph as presented to the conference by the International Labour Office (the addition of the words 'and calculation').

Later on in the same year the resolution quoted was discussed at a conference of the International Statistical Institute. That conference was not satisfied with the quoted paragraph and instead adopted the following paragraph 9:

"Un indice a le plus de valeur, lorsque, la qualité des articles dont les prix ont été relevés à des époques différentes et que les méthodes de centralisation et d'élaboration des données n'ont pas varié pendant toute la période visée."[2]

This, however, seems to be merely a different wording of the same thought.

At these conferences it was thus recognized that there exists a

[9] *Divisia* (1925), pp. 1129–30.
[1] *Conference* (1925), p. 70.
[2] *Bulletin* (1926), p. 127.

problem, but it was thought that the problem could be easily handled if the comparisons were restricted to identical articles. That the problem is more complicated than this was recognized by the Montreal Conference in 1947, which, though it did not treat the problem, stated its significance in a special resolution. It deserves to be quoted in full.

"The Sixth International Conference of Labour Statisticians,

Recognising that the measurement of price changes over time or of place-to-place differences in prices involves the establishment of equivalence in the quality of goods priced at different times or in different places at the same time, and

Recognising that cost-of-living indices may require the introduction of substitutions in the calculation of the index, or the adoption of such other methods as may improve the results, and

Recognising that methods should be devised for use when through the exigencies of fashion, new inventions, or for other reasons, the nature of the goods priced, designed for a more or less given purpose, has changed,

Requests the Governing Body of the International Labour Office to direct the Office to make a study of these problems, and to place the subject on the agenda of an early International Conference of Labour Statisticians."[3]

The resolution has so far not led to any action.

The importance of quality changes for price index comparisons was clearly recognized by several of the witnesses heard before the previously mentioned Subcommittee of the American House of Representatives.[4] Some specified references to these statements will be given further on, when different methods of attacking the problem have been indicated.

Sudden transition

Let us assume that at time o there is only one variety (*a*) of the item which we study, and that at time 1 this variety has disappeared and been substituted by variety *b*. As an example of such a transition from one variety to another we can cite the situation which arose when the fat content of milk was reduced in Sweden in the autumn of 1941. Previously milk sold for consumption had an average fat

[3] *Conference* (1948), p. 68.

[4] *Hearings* (1951); see especially pp. 50–51 (Hollander) pp. 72–84 (Hoover), pp. 175–176 (Smith), p. 183 (Cowden), pp. 229–234 and pp. 257–261 (Barkin), and p. 395 and p. 404 (Clague).

content of approximately 3.6 per cent and the price was 30 öre per liter (in Stockholm). From then on the fat content was standardized at 3.0 per cent and the price altered to 29 öre per liter.

To pay no regard to the quality change would be equivalent to using the quotient $\frac{p_1^b}{p_0^a}$ as a measure of the price development. In the sequel the term *average price method* will be used for this procedure. In the example this method would give $\frac{29}{30} = 0.97$, i.e. a 3 per cent price decrease. There are clearly strong objections to such a procedure.

In order to arrive at another result it must be possible to quantitatively compare the quality of the one article with that of the other. If the quality of article b is to the quality of article a as the constant g to 1, one would obtain

$$I_{01} = \frac{1}{g} \cdot \frac{p_1^b}{p_0^a} \tag{3.1}$$

This procedure was used for the official Swedish index in the above-mentioned situation for milk.[5] The caloric content was taken as a measure of the milk quality, and it was calculated that the higher fat content corresponded to 650 and the lower to 600 calories per liter. Consequently one obtained $g = \frac{600}{650} = 0.923$ and as a measure of the change in the price of milk $\frac{1}{0.923} \cdot \frac{29}{30} = 1.05$ was used, i.e. a 5 per cent price increase.

It might be mentioned that the fat content of milk was reduced in Denmark as well during the war. This was, however, not taken into consideration in the Danish cost-of-living index, where the prices were directly compared.[6]

[5] Formula (3.1) was thus, as a matter of fact, used for the Swedish cost-of-living index long before the author had anything to do with these calculations (from 1947 onwards). The formula is also used in *Frisch* (1943), p. 29, incidentally with the same notation g (the author did not see Frisch's book until this book was partly written). The significance of this formula has, however, not been generally recognized, and it is believed that this is the first time that it is given explicitly in connection with the computation of retail price indexes. In a popular fashion the formula was discussed in *Hofsten* (1949).

[6] According to oral information in 1948 from Mr. Ole Pedersen, then of the Statistical Department, Copenhagen.

It should at once be stressed that the selection of caloric content as a measure of milk quality is rather arbitrary. If some other property had been chosen instead, the result would have been different. The discussion of what should really be meant by quality will, however, be postponed until subsequent chapters. For the present it will be assumed that if an article is present in two or more varieties, unique numerical relations between their qualities can be obtained.

The transition is successive

In the example given the transition from one variety to another took place quite suddenly, from one day to the next. It is only in exceptional cases that a transition to a new variety takes place in this way. As a rule both varieties are for sale simultaneously, at least during a short period.

Let us assume that at time 0 only variety *a* can be bought. At time 1 variety *b* has been added to the market without variety *a*'s having as yet disappeared. At time 2 only variety *b* is to be bought. The object is to give the price development from time 0 to time 2.

In making a direct comparison from 0 to 2 we encounter the same difficulties as in the previous example. The quotient $\frac{p_2^b}{p_0^a}$ will not be a correct measure of the change. If we are in doubt as to the factor *g*, it is tempting to make the comparison in such a way that variety *a* is used between 0 and 1 and variety *b* between 1 and 2. By linking the two results one obtains

$$I_{02} = \frac{p_1^a}{p_0^a} \cdot \frac{p_2^b}{p_1^b} \tag{3.2}$$

In the sequel the term *the splicing method* is used for this procedure. The question now is whether the splicing method gives a result which can be accepted as a measure of the price change from 0 to 2.

In practice one usually has $p_0^a = p_1^a$ and $p_1^b = p_2^b$. The splicing method consequently gives $I_{02} = 1$. As an illustration let us assume that at one point in time (= 0) only one kind of orange is for sale and that its price is 1.50 kr per kg. At a later time (= 1) as well

oranges of a better quality are sold at a price of 1.75 kr per kg. After some time (at time 2) the oranges of the first kind have disappeared from the shops, but the oranges of the latter kind are still to be had at an unchanged price. The splicing method now gives the result 1, and it does not seem at first as though any objection could be raised against this result.

The condition that formula (3.2) give the same result as formula (3.1) is that

$$g = \frac{p_1^b}{p_1^a} \tag{3.3}$$

i.e. the quality relation between the varieties equals the quotient between the prices. One would imagine that in a free market prices would always adjust themselves so that (3.3) be fulfilled.

In the previous example with milk, the market was not free. The poorer quality could not have been brought into the market while the better was still there, unless the price difference had been much greater. At the given small price difference the transition must have been forced, i.e. the better quality must disappear before the poorer quality could be sold.

The most reasonable conclusion would seem to be that formula (3.1) should be used whenever the transition from one variety to another is enforced, a situation which occurs frequently during periods of war and rationing. Formula (3.2) in such cases will give a result which is too low. But, also, when the transition takes place voluntarily, formula (3.2) may give en erroneous result, viz. when the price in relation to the quality of the new article implies that it is cheaper than the old variety. In this case formula (3.2) will give a result which is too high, and formula (3.1) should again be used.

It is thus only when it can be assumed that the relation (3.3) is fulfilled that the splicing procedure (= formula 3.2) is legitimate. As will be demonstrated later, it cannot be taken for granted that this relation is in general fulfilled.

Several varieties for sale simultaneously

So far it has been assumed that only one variety—during a period of transition, two varieties—of an article are for sale simultaneously.

For many items a considerable number of varieties are for sale at one and the same time. Continual changes occur by the introduction of new varieties and the disappearance of older ones.

In the collection of price quotations for a retail price index it is impossible to distinguish between all different varieties and record a price for every variety separately. As has been pointed out, there may be hundreds of varieties of a single article.

In this situation there are two main ways out of the difficulty: one may either disregard the quality differences and base the index calculation on a comparison between average prices for all the different varieties (the average price method), or one may restrict the price quotations to one single variety, whose price development is then followed. In the latter case, if the selected variety disappears, one is forced to pass over to another variety and is then tempted to use the splicing method, i.e. the transition to the new variety will not affect the index. None of these procedures will in general give a result that can be accepted.

In order to illustrate the problem let us assume that on a certain day four different varieties of strawberries are for sale on a market. The price in kronor per liter for the different varieties and the development during the consecutive days are given in the following table.

Variety	Day number			
	1	2	3	4
A	1.25	1.25	1.25	1.50
B	1.75	1.75	—	2.00
C	2.00	—	2.25	2.25
D	2.25	2.25	2.50	2.50
Average price (unweighted)	*1.81*	*1.75*	*2.00*	*2.06*

According to the average price method a price reduction has taken place from day 1 to day 2. This is evidently not the case; the only thing that has happened is that variety C has disappeared, whereas the price of the other varieties is unchanged.

In some cases it may be possible to assign at least rough weights for the sales of the different varieties. However, this does not of course solve the problem. Assume, for instance, that the varieties

were sold in the proportions 1, 2, 3, 1 on the first day, and that on the second day the proportions were 2, 4, 2. The weighted averages will then be 1.86 kr and 1.75 kr. These figures do not give the price change from day 1 to day 2.

If one of the varieties *A*, *B*, or *D* had been chosen as a measure of the price development for strawberries, one would have found no price change from day 1 to day 2. Instead we assume that on day 1 variety *C* was chosen. When this variety disappears on day 2, one is forced to take another variety, e.g. *B*. It may be possible to reconstruct the price for variety *B* on day 1. Assume that one finds that no price change has taken place. The index will then be left unchanged. On day 3 variety *B* has disappeared whereas variety *C* has returned. In order to follow the price development from day 2 to day 3 one may turn to variety *A* and then finds no change. From day 3 to day 4 one may turn back to variety *C* and will then find no change.

In this way one will come to the conclusion that no change at all has taken place between day 1 and day 4. But as a matter of fact the prices of all four varieties have increased by 25 öre from day 1 to day 4, and an index should consequently show an increase of between 11 per cent $\left(\frac{25}{225}\right)$ and 20 per cent $\left(\frac{25}{125}\right)$.

It is of course possible to set up a rule according to which a direct comparison should be made, if a selected item disappears and then comes back again. In the example one would then have got $\frac{2.25}{2.00} = 1.13$. Such a rule may help in some cases, but it is of no aid for goods for which a successive quality change takes place and for which no return to the older varieties occurs.

Wireless sets may be chosen as an example. An application of the average price method is obviously out of the question. Such a series for the last 30 years would start with the crystal sets of the twenties and end with present day sets with loudspeakers, short wave bands and perhaps even gramophone attachments. Such a series would quite obviously not depict the price development for wireless sets.

But neither is the splicing method satisfactory. The price of a certain model of a certain make rarely undergoes any change. The

price changes occur when new models are put into the market. If prices are collected often enough, one can almost always find a model, which has maintained its price since the previous price collection. By continually changing models for price collection it is thus possible to construct an index which will always be unity. But this does not exclude the possibility that important price changes have taken place. Every new model may have been of a slightly higher quality than its predecessor but the price has increased a good deal.

Wireless sets may seem an extreme example, not typical of the goods which are included in a retail price index. However, in the Swedish cost-of-living index, clothing and miscellaneous items together represent almost half the total of the budget. Among these groups are to be found many industrial products, for which quality changes are frequent.

It should be noted that present-day industrial production works to a large extent with an unchanged price for a certain product. The system of resale price maintenance, which means that the producer fixes a price at which buyers of his product shall resell it, has become common. As long as a certain make, brand, or model is kept unchanged, the price will not be altered. When the producer wants to increase the price, he prefers to do it in such a way that he issues a new model, which may be only slightly different from the old one. It is not surprising that the producer finds this procedure preferable to increasing the price of the old product.

As a matter of fact, it turns out to be rather difficult to find good examples of goods which hold their quality both in the short and in the long run. Even among the food items, where the quality problems are least conspicuous, one finds that most items either have seasonal fluctuations (especially fruits and vegetables) or may change their quality in other ways, at least during war periods. The only example of a group of goods where quality changes are not too important seems to be certain kinds of fish.

What errors do the average price method and the splicing method give?

Let us now return to the case with only two goods, the one (a) being on the market at time 0, the other (b) at time 2, and both at

time 1. It is further assumed that the prices of a and b do not change, that the correct result is represented by formula (3.1), and that the factor g is known.

An error may be committed through the application of the average price method or the splicing method. The direction and magnitude of this error will depend upon how the new article differs from the old. Six different situations are possible. They are set out in the following table, in which I^K denotes the correct index and I^M an index according to the average price method. The splicing method always gives an index equal to unity.

Situation	Assumption	Result	Situation	Assumption	Result
A	$g > \frac{p^b}{p^a} > 1$	$I^M > 1 > I^K$	*D*	$g < \frac{p^b}{p^a} < 1$	$I^K > 1 > I^M$
B	$\frac{p^b}{p^a} > g > 1$	$I^M > I^K > 1$	*E*	$\frac{p^b}{p^a} < g < 1$	$1 > I^K > I^M$
C	$g > 1 > \frac{p^b}{p^a}$	$1 > I^M > I^K$	*F*	$g < 1 < \frac{p^b}{p^a}$	$I^K > I^M > 1$

It will be noted that $I^M > I^K$ in situations *A*, *B* and *C*, i.e. when $g > 1$, and that $I^M < I^K$ in situations *D*, *E*, *F*, i.e. when $g < 1$. This goes without saying; the interpretation is that if quality changes are not taken into account (average price method), the index will be too high when the quality has improved and too low when it has deteriorated.

We see again from the table that $I^K < 1$ in situations *A*, *C* and *E*, i.e. when $g > \frac{p^b}{p^a}$, and that $I^K > 1$ in situations *B*, *D* and *F*, i.e. when $g < \frac{p^b}{p^a}$. As splicing gives an index equal to unity, this means that this method gives too high a result for hidden price reductions, i.e. reductions which are disguised by a simultaneous change of quality. In the same way the result will be too low for hidden price increases.

Analysis of the six situations

Situation *A*. The price is higher than before, but the quality has increased more than the price. A correct index should consequently be less than unity. The average price method gives, however, an increased index. The splicing method, giving an unchanged index, is less incorrect.

This is a situation, which characterizes many comparisons between the war and the after war period. A suit, for instance, cost more in Sweden a piece in 1948 than in 1944, but was still cheaper in 1948 than in 1944 owing to the much better quality.

Situation *B*. Both price and quality are higher, but the price has increased more than the quality. A correct index should consequently be greater than unity, although not so great as the result obtained by the average price method. Splicing gives an unchanged index, i.e. the error obtained by that method is opposite in sign to that obtained by the average price method.

This is a situation which often occurs when a new model of an industrial article is put on the market. A new model of a wireless set, a car, a fountain pen is, as a rule, better than the older model, and if the price were the same the consumer would prefer the new model. It often happens, however, that the price of the new model is so much higher than the price of the old one that the transition to the new model implies a price increase.

Situation *C*. The quality is higher but the price lower. The average price method gives a somewhat lower index, but a correct calculation should give an greater decrease. The splicing method gives an unchanged index and is consequently further from the correct index.

This situation has not occurred so often since 1939. However, earlier it was not unusual for a new model of an industrial product to be definitely better than its predecessor and, notwithstanding that, to be cheaper.—Nylon products sold now may be of a better quality and, notwithstanding that, have lower price than the corresponding products sold a few years ago.

Situation *D*. Both the price and the quality are lower, but the quality has decreased more than the price. A correct index

should show an increase, but the average price method gives a lowered index. Splicing, giving an unchanged index, is less incorrect.

This situation can be illustrated by the milk example discussed above. It also happened on many other occasions during the war that substitutes were cheaper than the goods they replaced, but that the quality deterioration was greater than the decrease in price.

Situation *E*. Both price and quality are lower, but the price has decreased more than the quality. A correct index should consequently show a decrease, though not so marked a one as would be shown if the average price method were used. As splicing gives an unchanged index, the error then will have the opposite sign.

As an example of this situation may be mentioned the introduction of price control for shoes during the war. This action led to a considerably lower price and a somewhat lower quality.

Situation *F*. The price has increased, and at the same time the quality has decreased. A correct index should consequently show a marked increase, greater than the one obtained through the average price method. Splicing will give a still more erroneous result.

This is a situation which arose with many goods in the beginning of the war. In comparison with pre-war conditions one found that prices had increased and that simultaneously the quality had deteriorated (flour, sugar, clothes, coke etc.).

It must already be clear from this account that the frequency of the different situations varies between different periods. During a war period situations *D*, *E* and *F* must predominate, whereas situations *A*, *B* and *C* will be of greatest importance during a return to more normal conditions.

The accuracy of the definitions of the items

The results given by the index calculations will, to a considerable extent, depend upon the accuracy with which the index items are defined. If the selected articles are not accurately defined, the implication is that the average price method is used. This practice can always be criticized; if the index is to reflect the price development correctly, it is evident that price quotations for different goods must

not be directly compared. Otherwise the index will, as pointed out above, be too high when qualities increase and too low when qualities decrease.

But the more exactly one has defined the selected goods, the more often one will find that the article priced at time o will not be obtainable at subsequent times. One is then forced to make a transition to a new variety. If in such situations the splicing method is used, the consequence will be that the index will be too low for hidden price increases and too high for hidden price decreases.

If the average price method is used so long as the quality changes stay within the limits given by the specifications, but the splicing method employed while the quality changes fall outside the specifications given, the consequence will be that the result of the index calculation will depend upon the accuracy with which the goods have been defined. In the situations *A*, *B* and *F*, where $I^M > 1$, the index will be lower the more accurately the index items have been defined. In the situations *C*, *D* and *E*, where $I^M < 1$, it will be the reverse; the index will be higher, the more accurately the goods have been defined.

In the situations *B* and *E*, where I^K lies between I^M and 1, a combined use of the average price method and the splicing method may under favourable circumstances give a correct result. But in situations *A*, *C*, *D* and *F*, where I^K is either greater or less than both I^M and 1, this is evidently not the case.

The relation between linking and splicing

This relation should be quite clear from what has been said above, but as these two procedures may easily be confused with each other, it seems appropriate to discuss the matter somewhat.

The chain index (linking) means that a new index is computed for every new link. If Laspeyres' formula or one of the other similar formulas are used, this means that the q-values for the different budget items are exchanged for every new link. In principle the procedure postulates that the goods are the same at the beginning and the end of every link or that changes in the totality of goods are taken into consideration.

The splicing method implies that when a commodity, selected as a price quotation article, has disappeared or is about to disappear, it is replaced by another article. If the chain index method is used, it is of course very easy to exchange the items in this way. Splicing may, however, well be applied also when an index series is based on fixed weights.

The important distinction between linking and splicing was not clear for the Swedish Index Committee of 1942. While recommending a transition from fixed base index to chain index it stated that "it should to a large extent be possible to overcome the inconvenience for the price quotation, which is connected with variations in quality and supply of the selected goods, by the application of the chain method, recommended above" ("den olägenhet för prisnoteringen, som sammanhänger med växlingar i kvalitet och tillgång på noteringsvarorna, bör i stor utsträckning kunna övervinnas genom tilllämpning av det i det föregående rekommenderade kedjeförfarandet"[7]). As has been indicated above, this is not the case.

Some additional references to literature

The chain index method has been criticized by Keynes, who has especially stressed the risk that, if this method is used, small successive errors of approximation of a cumulative nature are neglected. Among other things he says that "the Chain Method will show a cumulative error if each substitution which we assume as approximately equivalent is progressively a slight improvement, i.e. if each new composite is slightly better for the purpose in view than its predecessor, and is not just as likely to be slightly worse."[8]

Keynes then does not make any distinction between linking and splicing, but imagines that the chain method *per se* implies splicing. This need not be the case; as pointed out above a correct application of the chain method implies that consequences of the variations in the totality of goods must be taken into account for every link.

When it comes to comparisons over a longer period of time Keynes recommends "the Highest Common Factor Method supplemented by

[7] *Betänkande* (1943), p. 28.
[8] *Keynes* (1930), p. 118.

the Method of Equivalent Substitution and the Method of Limits."[9] Briefly stated, this implies that the comparison should mainly be restricted to those parts of the consumption, which are identical at the beginning and at the end of the period. Keynes assumes that a considerable part of the consumption is common, even if the comparison refers to a long period. Thus he says:

"I conclude, therefore, that it would be better in comparing, let us say, the purchasing power of money to-day and fifty years ago to compare the price of that part of expenditure (say 50–70 per cent—I have no idea as to the true proportion) which is more or less unchanged in character, supplementing this by a list of expenditures discarded and added (so as to enable a general judgment to be made as to the extent of improved opportunities)."[1]

As a matter of fact only a *very* small proportion of the goods on the retail market are the same now as they were fifty years ago. During that period practically all goods have undergone important quality changes. A comparison which refers only to the very few identical goods may be quite misleading. A general judgment for the remaining and, in point of fact, quite dominating part of the consumption will be very arbitrary.

Ulmer does not seem to share Keynes' opinion that the substitution of goods in the index may lead to errors. He points out that all changes in the totality of goods—what he calls environment—are, except during periods of war, gradual.[2] When a new article enters the market, it may often at first sight be regarded as a luxury article, which should not influence the index.

"It is during the second, or competitive stage, that action may be profitably taken by the computer of cost of living indexes. It appears reasonable to believe, insofar as this normal pattern is followed, that as the product is improved and its price lowered a point may be reached at which it appears that for the mass of consumers a choice between the new product and the old one at the prices then prevailing has reached a condition approaching indifference.———Hence, *at that point in time*, it would be justified to link the new commodity into the index at the same price level as the old one."[3]

[9] *Keynes* (1930), p. 119.
[1] *Keynes* (1930), p. 119.
[2] *Ulmer* (1949), p. 65.
[3] *Ulmer* (1949), pp. 67–68.

Ulmer is thus of the opinion that splicing is permissible, provided that it takes place at the correct moment. It is true that he also mentions cases in which splicing should not be undertaken, but he seems to find such situations less important and gives no method for these cases.

In a report prepared for the Sixth international conference of labour statisticians the International Labour Office has stated that it is "of fundamental importance" that the prices "relate to goods of the same or equal quality."[4] Further on the office says:

> "A special problem arises where the articles change from one month to the next. In cases where the goods priced in the preceding month are not available for pricing during the current month, the common practice is to obtain prices for the preceding month as well as for the current month for a similar item, so that the price changes reported relate to identical articles."[5]

Splicing is thus described and it is considered legitimate except during periods of war.

An author which should also be mentioned here is Frisch. In the calculation of an index for building costs he has treated the quality problem.[6] In the main Frisch is of the opinion that splicing is permissible, but he also mentions situations where a direct quality comparison should be undertaken. However, when it comes to the computation of an index for production costs, the problems differ so fundamentally from the problems encountered in the construction of a retail price index that there is no reason to consider further Frisch's points of view.

Reference may here also be given to another ILO report in which quality problems in connection with measurements of production volumes are treated, though only briefly.[7] Quality problems from the point of view of the producer are also treated in a recent study by Brems.[8]

[4] *Cost-of-living* (1947), p. 20.
[5] *Cost-of-living* (1947), p. 21.
[6] *Frisch* (1943).
[7] *Productivity* (1951).
[8] *Brems* (1951).

The procedure for the American index

Attention has been paid to the quality problem in connection with the computation of the American Consumers' Price Index long before the above-mentioned hearings took place in 1951. Much effort has been made to find the methods which seemed to be in accordance with the general purpose of the index. It is consequently of interest to give an account of the procedure.

An elaborate set of detailed specifications of the articles to be included in the index has been prepared for the use of the agents who collect the price data. As an example the following specification for overalls may be quoted:

"Overalls, white-backed denim. Fabric: cotton, 2.2 yards per pound before sanforizing, sanforized to approximately 8 ounces per yard. Pockets of drill, sheeting, or similar fabric, sanforized shrunk. Construction and styling: 45 to 46 yards per dozen, based on 28-inch fabric and 32 to 42 size scale, high back, bib style, double-stitched seams, bartacked at points of strain. Minimum amount of trimming. Specify with or without union label."[9]

Even within such narrow specifications different varieties may be found. Therefore, the price collectors are instructed to select identical goods within the given specifications. For the transition to new varieties the following rules are followed.

"Prices are obtained for identical articles as long as they are available in retail stores. When the Bureau's representatives can no longer obtain prices for a given article, they must substitute another.

Substitutions are of two types: (1) Substitution of another article which is adequately described by the same specification, and (2) substitution of an article serving the same purpose, but not of the same quality, and described by a new specification.

In the first type, any difference in price between the old and new article is shown as a price change in the index calculation. For example, if one brand of men's shirts is no longer available and another brand of substantially the same quality is substituted, the difference in price is allowed to affect the level of the index. In the second type of substitution, the level of the index is not affected, for the new article is introduced by a linking process. An example of this type of substitution is the replacement of silk hose by rayon hose during World War II. Substitute specifications are

[9] *Hearings* (1951), p. 232.

always made to adhere as closely as possible to those supplanted, i.e., with respect to utility of goods, materials, designs, and price movements."[1]

This account is completed by the following information, given by Edward D. Hollander, Chief of the Division of Prices and Cost of Living at the Bureau of Labor Statistics:

"Within each specification used by the Bureau there is some variation in quality. Under regular procedure, prices of substitute articles meeting the same specifications are compared directly with the prices of the articles previously priced and any price difference allowed to affect the index; however, prices of articles not meeting specifications, or meeting revised specifications are linked into the index.

For many commodities, such as automobiles, electric refrigerators and other consumer durable goods, price changes almost always occur simultaneously with quality changes, usually at the time of introduction of new models. Prices of the new articles are compared directly with those of the old models and the index reflects the increase in absolute price without adjustment for the improved quality.

During the war period when quality deterioration and price changes effected by means of quality deterioration were so prevalent in this country, it was felt that all quality changes were in one direction. The Bureau, therefore, departed from its regular procedure to some extent. When a *lower* price was reported for goods substituted for a discontinued line, prices were linked so as to show no decrease in price. However, for goods introduced into the index at the *same* or *higher* price than discontinued goods of superior quality, it was possible to reflect in the index only the increase in absolute price.

In some cases when goods of prewar quality became generally unavailable, revised specifications were written to cover wartime substitutes, and current prices of these substitutes were not linked as in regular procedure, but were compared directly with prices of prewar specifications, unless they were superior in quality or entirely dissimilar. In a further effort to measure the cost to the consumer of disappearance of lower quality goods, where the Bureau priced several qualities of an article and the lowest grade disappeared from retail stores, the price of this quality was compared with the current prices of the next higher quality."[2]

The splicing method is thus only used for non-durables, when articles meeting the given specification can no longer be obtained. During the war some deviations from this procedure took place. The

[1] *Monthly Labor Review,* Sept. 1949, p. 285.
[2] In a letter to the author, dated November 15, 1949.

average price method is generally used for durables and for other goods for which the specifications are fulfilled.

Additional information on the details of the procedure can be obtained from the record of the Congress hearings in 1951.[3] At these hearings the splicing procedure, as utilized by the Bureau of Labor Statistics, was criticized by several witnesses, and among them especially Salomon Barkin, research director at the Textile Workers Union of America.[4] In his response to this criticism, which to a considerable extent coincides with the views expressed earlier in this chapter[5], Ewan Clague made it clear that the endeavour of the Bureau of Labor Statistics is to base the comparison on prices for identical articles, and that when this cannot be done splicing is considered appropriate. To judge from his account splicing is used more than would be believed from the quotations above. The following passage from a written statement submitted to the Subcommittee is illuminating:

"Our general policies are clear. In the first place, we try as long as we can to price the identical article, since price change in the absence of quality change is the simplest to measure, for example, 92 score butter. On the other hand, when commodities are not standardized, like women's clothes, the pricing is more complex, and we need a way of shifting from one item to another without confusing quality change and price change. When we shift our pricing of an article from one quality to another, where they are both in the market at the same time, we assume that the difference in price is the measure of difference in quality, and make the change in such a way that the difference in price between the article being dropped from pricing does not affect the level of the index."[6]

In this statement it is thus recognized that splicing implies the assumption that the relation (3.3) is fulfilled.

Although the procedure followed was defended by Clague and other officials of the Bureau of Labor Statistics it was also admitted that it might lead to erroneous results, especially during periods "when goods are not freely available in the market, or when, for other

[3] See *Hearings* (1951), especially pp. 81–84.
[4] *Hearings* (1951), p. 209, p. 234, and pp. 258–261.
[5] The text of these hearings did not reach the writer until this book was nearly completed.
[6] *Hearings* (1951), p. 395.

reasons, the market is not free to place its values on various goods."[7] And the Subcommittee in its report stated that

"we, therefore, urge that the Bureau make all possible efforts, perhaps in cooperation with organizations such as the Bureau of Standards, to develop techniques which will more adequately reflect the quality changes which take place in the goods priced for the index."[8]

[7] *Hearings* (1951), p. 395 (Clague).
[8] *Report* (1951), p. 38.

Chapter 4.—Some Numerical Results from the Swedish Index Computations[1]

The construction of the Swedish cost-of-living index

To start with the Swedish cost-of-living index was a fixed base index based upon Laspeyres' formula. The quantities were derived from a household budget enquiry referring to 1913–14, and the index aimed at showing the price development after July 1914.

In 1932 a revision of the index took place. The items to be priced were partly revised and new quantities, derived from a budget enquiry in 1923, were introduced. In this way a quite new series for which 1931 was chosen as the base year was started, but this series was linked to the older one, and as published the index still referred to 1914 = 100.

In 1939 the results from a new budget enquiry in 1933 had become available, and the quantities were again revised. This time the 'basis of publication' was altered but to 1935 and not to 1933. The old series, in which 1914 = 100, was continued as an alternative.

From 1943 onwards the budget has—at least in principle—been revised once a year, and the index is thus a chain index with yearly links, which are computed according to formula (1.11). During every year the weights have, as a rule, been kept constant. In the war years, when new rationing schemes were introduced or when rations were altered, it also occurred that the weighting diagram was revised during the year. The results obtained for the yearly links are

[1] The figures given in this chapter have been worked out by the Social Welfare Board on the author's initiative. He is also responsible for the special investigations undertaken in 1948 and for the proposal to change the procedure in 1950.

multiplied, and as published the index refers only to 1935 (and 1914).

Simultaneously with the introduction of the yearly revision of the weights the splicing method was introduced for the separate items in the index. In practice the splicing procedure which, as a general rule, was abolished in 1950, took place in the following way.

Prices are collected every quarter[2] from 25 towns and other urban communities (food and fuel items from 60 places). Special agents, representing the Social Welfare Board, which is responsible for the index computations, regularly visit a number of shops. As a rule only one price is collected from every retail outlet. For a number of items it has been found more reliable and practical to collect the prices centrally (margarine, cocoa, electric current, sewing thread, torch batteries, tobacco etc.). Altogether, prices for 85 items are collected in this way. As mentioned earlier, the total number of items is 236.

The details of articles for which prices are collected by the agents are entered upon special forms. The specifications of quality etc. on these forms are reasonably detailed, but for many items it is nevertheless possible to find different articles with quite different prices, fulfilling the given specification. The agents are instructed that they should, as far as possible, give prices for the same article one quarter after the other, and to note if they have had to transfer to a new article.

On arrival at the Social Welfare Board the completed forms are compared with the corresponding forms received one quarter earlier. From 1943 to 1949, as soon as it was thereby discovered that an actual price quotation did not refer to the same quality as last quarter, these quotations (the new and the old) were deleted, and the comparisons were made only with identical articles.

In 1950 this procedure was abolished. The index is now in principle based on the prices received from the agents, not taking into

[2] As a matter of fact most prices are collected every month. The monthly figures are, however, only used for the Consumers' price index. Only the cost-of-living index, which is generally considered to be of greater importance, is dealt with here. It is only computed once a quarter.

account if these prices do not refer to exactly the same articles, provided only that the selected article meets the specification.

As the specifications are less detailed than the corresponding specifications for the American index (cf. end of previous chapter), it does not so often occur that price quotations cannot be obtained for a desired article. When it is necessary to alter the specification, a judgment is made from case to case as to whether splicing or averaging or some intermediate shall be applied. It is for the centrally priced items, for which the specifications are most accurate and for which not so many articles are available simultaneously, that specifications have to be altered most frequently.

The difference between the results obtained through splicing and through averaging

For a number of items the index obtained through the splicing procedure is quite different from the index which is obtained if averages of the prices received from the agents are directly compared. As an example the following table shows the price development for coffee-cups from December 1946 to December 1949.

Month	Average of collected prices, Kr. per dozen	Index based on average prices	Index with splicing
December 1946	9.09	100.0	100.0
March 1947	8.73	96.0	95.2
June	8.75	96.3	95.2
September	10.24	112.7	95.2
December	9.86	108.5	95.3
March 1948	10.52	115.7	95.5
June	12.00	132.0	95.6
September	14.02	154.2	113.6
December	15.87	174.6	115.3
March 1949	15.44	169.9	116.1
June	15.26	167.9	116.1
September	14.88	163.7	116.1
December	14.74	162.2	116.2

Here the index with splicing is the one actually used for the index calculations during the period in question. For the whole period it gave a price increase of 16.2 per cent. An index based on the unrevised figures gives a completely different result. According to this series the price has risen 62.2 per cent during the same period of three years.

During the period in question there was a general tendency to hidden price increases, especially for articles which were in short supply, as in the case of coffee-cups (cf. below). There is consequently no doubt that the splicing method has given too low a result.

On the other hand, the period from 1946 to 1949 was characterized by improvement in quality. Increased real incomes made it possible for the public to buy better articles at the end of the period than at the beginning. This tendency, and the fact that the scarcity was most marked in the cheaper articles, may have caused the agents to give prices for articles of a better quality in 1949 than in 1946. Although nothing definite can be said, the conclusion seems to be valid that the index based on the unrevised prices received from the agents is too high.

A correct index for December 1949 should then show a result between 116.2 and 162.2. The trouble is that the interval is such a wide one. From the figures given it is impossible to state what the 'correct' figure should be.

This is an extreme example. Only a few articles, like coffee-cups, show such a great discrepancy between the average price and the splicing methods. But some discrepancy is found for most articles in the 'clothing' group, and for many articles in the 'miscellaneous items' group in the index. As these two groups, as already mentioned, together account for nearly half the budget of the index, the problem is of great importance. Nor is the problem negligible for the remaining part of the budget, including 'food and drink,' 'fuel and light,' and 'housing'.

A special investigation in 1948

Some more light is thrown on the problem under discussion by the results of a special investigation undertaken in the autumn of 1948.

The object of this investigation was to study the shortage of certain articles, viz. coffee-cups, glasses, shirts, and chairs.

The price agents were instructed that they—besides their regular price collection—should quote prices for *all* varieties meeting the specifications for the articles mentioned. The same shops as usual should be visited, and for September and November results were obtained from three shops in each of 24 places, i.e. 72 shops in all. For coffee-cups the number of quotations per shop is seen in the following table.

Number of quotations	Frequency Sept. 48	Nov. 48
0	30	37
1	13	10
2	14	11
3	9	15
4	6	1
5	2	1
6	—	1
Number of shops	72	72
Number of quotations	102	92

There was thus a marked shortage. That is, however, not what is of primary interest here.

The specification for the article in question is the following: "Coffee-cups with saucers, bone-coloured glazed ware with simple decoration, one dozen." This specification may seem accurate enough, but it was nevertheless possible to find such different prices that the highest price is almost four times as high as the lowest one. This is seen from the table on the next page.

The disparity between prices as shown in this table, is to some extent due to real differences from place to place. However, quite different prices were also obtained in the same shop or in different shops in the same place.

There are, of course, a great number of articles for which this disparity is not at all so great. However, for most articles there is at least some such difference in price, and the number of articles for

Price, Kr.	Number of quotations September 1948	Number of quotations November 1948
7.50— 9.49	5	1
9.50—11.49	3	—
11.50—13.49	18	20
13.50—15.49	27	29
15.50—17.49	5	5
17.50—19.49	17	16
19.50—21.49	17	13
21.50—23.49	2	2
23.50—25.49	2	3
25.50—27.49	5	2
27.50—29.49	1	1
Total	102	92
Average price, Kr.	*15.13*	*15.49*
Index (September = 100)	100.0	102.3

which it is appreciable is by no means small. It is of course impossible to exclude all such items from the index calculations.

It is of interest to compare the averages obtained in the special investigation with the corresponding figures obtained from the regular price collection. This is done in the following table.

Article	Regular price collection				Special investigation		
	Average price, Kr.		Average index	Spliced index	Average price, Kr.		Index Sept. = = 100
	Sept. 48	Dec. 48	Sept. = 100		Sept. 48	Nov. 48	
Coffee-cups	14.02	15.87	113.2	101.5	15.13	15.49	102.4
Glasses	5.39	5.73	106.3	102.9	6.82	5.92	86.8
Shirts	15.66	15.68	100.1	100.4	17.94	17.29	96.4
Chairs	48.59	48.99	100.8	100.0	53.96	52.68	97.6

As a whole the averages obtained from the special investigation are higher than the corresponding figures obtained from the regular price collection. This is not surprising; the agents tend usually to select articles of simple design etc.

What method should be used?

As shown in the previous chapter, a comparison which is based on a selection of identical articles, i.e. splicing, may give a completely erroneous result.

It is also clear that a direct comparison of the averages for a limited number of articles, selected by the agent, may be a dangerous method. If the agents tend to select better and better articles, the index will become too high and vice versa.

A direct comparison of the price averages for all articles meeting the specification may at first sight seem to be a good method. But this method may also give erroneous results. The disappearance from the market of the more expensive articles within the specification will, for instance, result in a decrease of the index which is not acceptable. From the table given above it can be seen that the index which is obtained in this way may act very capriciously. For example, it would have been hard to justify the price decrease for drinking glasses obtained in this way from September to November 1948.

None of the three methods is satisfactory, then. In order to get a correct index one must a) be able to distinguish between all different articles, b) obtain prices for each article separately, and c) have measures of the quality relation between the articles. For many items this is out of question. In practice one is forced to rely upon the three methods given above, i.e. 1) identical articles and splicing; 2) comparison of averages for articles selected by agents, and 3) comparison of averages for all articles meeting the specifications.

Of these, splicing seems to be the least satisfactory. In the long run especially, this method may for many items give a result which can under no circumstances be accepted. If the articles are accurately defined the two other methods must give a reasonably good result, so long as articles within the given specification can be obtained. In order to avoid the capricious results which will be obtained if prices are collected for all articles meeting the giving specification, it seems better to let the agents select the articles, for which they give prices, and to accept averages based on these prices for the

index computations. As stated above, this is the procedure which is now (since 1950) followed for the Swedish index calculations.

In this way an index series will be obtained for every item. At regular intervals—say, every five years; more often in case of more frequent disturbances—these series should be checked for all articles for which the price disparity between different varieties is great. Through special investigations, covering all the varieties on the market, it should be possible to ascertain whether the price indexes for the separate articles can be accepted.

The more accurate the specifications for the separate items, the more often will one be forced to alter the specification. Whenever this has to be done, one must decide whether the new article shall be spliced into the index or not. If the two articles have both been present on the market for a fairly long period, and no important changes in their price relation have taken place during this period, splicing may take place. But if there was a shortage of the old article when the new article was introduced or if there were obvious changes in their relative prices, splicing should not be employed. It is then necessary to estimate the quality of the new article in relation to the old, i.e. formula (3.1) should be employed.

A recomputation of the Swedish cost-of-living index

In order to obtain some information on the difference between the all items index based on splicing and that based on the average price method, a recomputation of the Swedish cost-of-living index from December 1946 to December 1949 has been undertaken. Before 1946 rationing and other war-time phenomena dominate the picture, and after 1949 the splicing procedure was abolished as a general rule for the official index, so the chosen period is the only one which is suitable for the purpose.

The result of the recomputation is set out in tables 4.1—4.4. In these tables the average prices obtained at the beginning and at the end of the period are shown, and also the figures obtained by division of these averages. Further on, the tables show the index figures actually used, i.e. the figures based upon splicing. Finally, they give the cost-weights actually used for the official index from December 1946 to December 1947.

In a number of cases the specifications of the articles have been altered during the period. In such cases the average prices have not been divided by one another; instead, splicing at the point of transition has been performed. These cases are specified in the remarks to the tables.

It will be seen from the tables that the average price method gives the same, or practically the same, result as the splicing method for a good many items. This means, of course, that splicing of the individual price quotations has not taken place or has been of minor importance. However, for other items the direct comparison of the averages gives a result which deviates considerably from the result obtained through splicing. This is notably the case with some clothing articles (mens' suits and overcoats, shirts, rayon stockings), further with shoes and with some household articles (china, glasses etc.).

Items which were included in the index for only part of the period, are not shown in tables 4.1—4.4, and an exact reconstruction of the actual index is consequently not possible on the basis of these tables. It would also be a little complicated to demonstrate the yearly variations of the weights, which have been undertaken.

Weighted averages of the index figures in the tables 4.1—4.4 can, however, easily be formed, using the weights in the right hand columns of the tables. The result of this computation is shown in the following table in which also the result of the official index calculation is given.

For 'rent' and 'fuel and light' it has been necessary to use the official figures also for the recomputation. For house rents the reason is that a recomputation cannot be performed, as the official index is based upon housing censuses as 'benchmark' data, and the yearly revision of the rent index is based on estimates. Different rationing schemes during the period would make a recomputation of the fuel and light index rather complicated.

From the following table it is seen that the recomputation based on splicing gives almost the same result as the official index. The combined effect of the yearly revision of the weights, the exclusion of a number of old items and the inclusion of a number of new items has only been to lower the all items index from 106.6 to 106.2. The corresponding differences for the separate item groups are also small and insignificant. The figures based on the average prices are, on the

Item group	Recomputation with averaging	Recomputation with splicing	Official index calculation	Weight, Kr., December 1946
Food	109.4	109.5	109.1[3]	2406
Rent	.	.	103.6	634
Fuel and light . . .	.	.	101.9	306
Clothing	115.4	101.7	101.5	829
Shoes	102.0	99.6	98.5	182
Miscellaneous	110.3	108.0	107.1	1326
All items	*109.2*	*106.6*	*106.2*[3]	*5683*

other hand, definitely higher, except for food where the figures are practically the same.

As stated above it may be assumed that for the period from 1946 to 1949 splicing has given figures which are on the low side. On the other hand, owing to the fact that the quality of many articles was better at the end than at the beginning of the period, the recomputation based on the average prices should be considered to give a result which is somewhat too high. The conclusion would be that the 'correct' index falls between 106 and 109.

However, these bounds are widely separated, remarkably so. It should also be realized that both the splicing and the average price method may be applied in other ways than has been done here. More accurate definitions of the price quotation items would have resulted in a still lower splice-index figure. Similarly, less accurate definitions may have resulted in a higher average price index.

Finally it should also be remembered that the arguments refer only to one specific period. It cannot in general be assumed that the 'correct' index is included between the results obtained through splicing and averaging (cf. chapter 3). The only definite conclusion to be drawn from the material presented here seems to be that the interpretation of the price development for the different items constitutes a problem which is of much greater numerical importance than the selection and the weighting of the items.

[3] The official index as a matter of fact gave 113.1 for food and 108.0 for all items for the period in question. This is due to the fact that a system of price rebates on milk, butter, and margarine for certain categories of the population, which was abolished on January 1st 1948, was taken into account for the official index calculations but not for the recomputations made here.

Table 4.1.—*Food*

Item	Quantity	Average price, öre		Index of average prices	Index figure actually used	Weight, Kr., Dec. 1946
		Dec. 1946	Dec. 1949			
Milk, full cream . .	l	30.2	33.6	111.3	111.3	301
Cheese, whole fat .	kg	304.5	320.0	105.1	105.1	74
Dairy butter	„	455.0	560.1	123.1	123.1	208
Margarine (c) . . .	„	270.0	260.0	96.3	96.3	36
Eggs, new-laid . . .	„	384.4	342.1	89.0	89.0	123
Meat:						
joint for roasting .	„	324.8	424.3	130.6	130.6	45
„ „ soup . .	„	263.1	289.6	110.1	110.1	54
minced	„	301.3	359.7	119.4	119.4	85
Veal:						
joint for roasting .	„	430.5	613.1	142.4	142.4	24
shoulder	„	378.1	471.4	124.7	124.7	26
Mutton:						
joint for roasting .	„	468.5	500.1	106.7	106.7	16
Fowl	„	365.9	347.0	94.8	94.8	8
Pork chop	„	464.4	443.2	95.4	95.4	17
Salted pork	„	396.7	344.4	86.8	86.8	36
Bacon	„	600.2	683.5	113.9	113.9	9
Boiled ham	„	383.9	449.9	117.2	117.2	73
Pork-sausage . . .	„	258.5	251.2	97.2	97.2	64
Breakfast sausage .	„	293.6	305.2	104.0	104.0	64
Black pudding . . .	„	124.2	123.7	99.6	99.6	21
Fish-balls	pot	76.1	83.0	109.1	109.1	9
Fresh fish:						
Herring	kg	131.0	108.2	82.6	82.3	13
Sprats (Baltic herring)	„	126.5	126.6	100.1	100.1	13
Mackerel	„	186.9	169.4	90.6	90.1	5
Perch	„	186.5	184.8	99.1	98.8	6
Pike	„	319.9	329.5	103.0	103.3	6
Cod	„	179.5	182.2	101.5	97.3	22
Haddock	„	172.1	230.4	133.9	130.5	6
Salt herring	„	176.3	152.3	86.4	84.7	38
Anchovy	pot	126.5	118.1	93.4	93.7	16
Plain flour	2½ kg	130.3	170.9	131.2	131.2	64
Whole flour	„	107.6	144.7	134.5	134.6	15
Rolled oats	kg	69.2	74.7	107.9	107.1	19

Table 4.1.—*Food* (cont.)

Item	Quantity	Average price, öre		Index of average prices	Index figure actually used	Weight, Kr., Dec. 1946
		Dec. 1946	Dec. 1949			
Macaroni	kg	84.3	103.2	122.4	120.9	11
Crisp bread, coarse	„	88.7	110.4	124.5	118.4	18
„ „ , fine	„	115.1	128.8	111.9	113.4	18
Brown bread	„	75.0	85.7	114.3	114.3	36
Wholemeal bread	„	63.4	62.9	99.2	109.4	27
Wheat rusks	„	204.0	234.4	114.9	114.8	16
White bread: made with water	„	94.7	111.0	117.2	117.3	15
„ „ milk (milkloaf)	„	104.4	110.5	105.8	105.0	7
Biscuits	„	27.0	29.0	107.4	107.4	16
Buns	„	154.3	187.9	121.8	119.6	52
Danish buns	„	280.7	399.3	108.3 (142.3)	110.7	32
Coffee	„	541.3	579.7	107.1	104.4	96
Cocoa (c)	„	403.0	412.0	102.2	102.2	14
Granulated sugar	„	61.8	80.6	130.4	130.4	75
Yeast	hg	16.2	15.8	97.5	97.5	18
Dried peas	kg	63.4	74.6	117.7	117.7	8
Tinned peas	½ pot	85.8	80.8	94.2	93.3	12
Potatoes	kg	21.9	23.7	108.2	118.5	92
Carrots	„	36.3	44.9	123.7	123.6	5
Swedes	„	25.5	30.4	119.2	119.1	5
Cabbage	„	36.1	40.0	110.8	110.5	9
Onions	„	99.4	75.3	75.8	75.8	10
Tomatoes, Swedish	„	164.0	189.9	115.8	106.0	8
Spinach	„	188.0	198.5	105.6	106.6	6
Red currants	„	69.7	107.1	153.7	151.3	2
Cranberries	„	88.6	112.0	126.4	126.4	21
Apples, Swedish	„	181.2	167.1	92.2	91.1	90
Oranges	„	130.5	125.1	95.9	95.9	42
Lemons	„	197.5	174.7	88.5	88.5	2
Prunes	„	185.8	153.4	82.6	82.5	39
Bananas	„	258.5	243.5	94.2	94.2	20
Beer	⅓ l	31.5	33.3	105.7	105.7	16
Small beer	5 l	131.6	135.9	103.3	103.5	10
Lemonade	⅓ l	25.1	27.9	111.2	111.2	10
Restaurant meal (c)	—	.	.	117.0	117.0	16
Sweets (c)	kg	560.0	620.0	110.7	110.7	7
Chocolate toffee (c)	„	641.0	738.4	115.2	115.2	9
All food items				*109.4*	*109.5*	*2406*

Remarks to the tables 4.1–4.4.

The weights given in the tables approximate the cost-weights in kronor actually applying during the period December 1946–December 1947.

The letter c denotes items for which prices were collected centrally; for other items prices were collected by the agents.

Items which were only included in the official index computation for part of the period do not appear in the tables. Thus 7 food items representing 2 per cent of the food budget in 1946–47 had been excluded before December 1949. For clothing 4 items representing 6 per cent and for miscellaneous 7 items representing 12 per cent were excluded. In a similar way 13 new food items representing 13 per cent of the food budget had been included before December 1949. For clothing there were 10 new items representing 30 per cent, and for miscellaneous 6 new items representing 9 per cent of the budget.

For a number of items no prices for defined articles have been collected; instead, the index calculations for the items in question have been based on special analyses which have been undertaken from time to time. In these cases no figures are given in the left hand columns of the tables. For such items the index figures actually used have been employed also for the recomputed index otherwise based upon average prices.

In some cases the specification of the article has been altered during the period in question. In such cases the index obtained by division of the average prices is given in parenthesis, whereas the index figure obtained by splicing at the point of transition is given before the parenthesis and is employed for the present recomputation. The manner in which the altered specification has taken place is indicated in the following comments to the separate tables. At the same time some other remarks are also given.

Table 4.1.—*Food*

For *fresh fish* previous to October 1948 prices had been collected three times a month; from then on prices were only collected once a month. The difference which arose in this way was not taken into account for the regular price index computations.

Danish buns: To March 1949 simpler quality ("snäckbröd"); from then on better quality ("vinerbröd").

Potatoes, tomatoes, spinach, red currants, cranberries: For the regular index calculations seasonally adjusted figures have been used. The average prices refer to the last month of the year when these articles were plentiful, viz. for potatoes December, for tomatoes, spinach and cranberries September, and for red currants August.

Table 4.2.—*Clothing*

Item	Quantity	Average price, Kr.		Index of average prices	Index actually used	Weight, Kr., Dec. 1946
		Dec. 1946	Dec. 1949			
Man's suit	one	141.99	184.71	130.1	101.3	233
Man's overcoat . .	„	136.18	163.75	120.2	100.2	85
Trenchcoat	„	102.49	113.76	111.0		43
Leather jacket . . .	„	88.00	97.00	110.2	110.2	26
Shirt, poplin	„	12.85	16.49	128.3	98.9	60
Workman's shirt . .	„	6.67	10.62	112.7 (159.2)	112.3	11
Overall	„	14.81	18.05	121.9	122.2	10
Drill shirt	„	6.86	8.55	124.6	124.3	10
Hat	„	17.06	28.52	94.5 (167.2)	94.5	20
Cap	„	6.96	8.47	104.1 (121.7)	97.6	20
Man's gloves . . .	pair	16.05	15.88	98.9	96.5	10
Dress material:						
woollen	meter	13.65	14.22	104.2	104.2	112
cotton	„	1.65	1.69	102.4	102.4	22
fibre mixture (cell-ull)	„	2.11	2.36	111.8	103.1	14
Pants	pair	4.79	5.78	120.7	117.9	16
Vest and knickers (c)	„	17.25	17.50	101.4	101.4	11
Baby's knickers (c) .	„	2.78	2.00	71.9	71.9	6
Stockings, rayon . .	„	4.80	5.27	109.8	96.4	22
Man's socks	„	4.24	3.92	92.5	92.5	28
Children's socks . .	„	1.25	1.68	134.4	132.6	12
Woollen oversocks .	„	5.07	3.55	97.4 (70.0)	97.4	14
Knitting wool . . .	kg	19.00	22.16	93.4 (116.6)	93.4	32
Sewing silk (c) . . .	50 meter	0.37	0.25	67.6	67.6	4
Sewing cotton (c) . .	200 yards	0.45	0.50	111.1	111.1	8
All clothing items .				*115.4*	*101.7*	*829*

Table 4.2.—*Clothing*

Man's suit, man's overcoat and trenchcoat. For these items the prices collected from the agents were not used during this period. Instead, the

Table 4.3.—*Shoes*

Item	Average price, Kr. per pair		Index of average prices	Index actually used	Weight, Kr., Dec. 1946
	Dec. 1946	Dec. 1949			
Man's shoes:					
a) price-controlled	28.25	27.75	98.2	97.1	7
b) free	31.61	36.91	116.8	97.1	7
Man's boots, price-controlled	25.18	25.00	99.3	97.0	10
Ladies' shoes:					
a) Oxford, free	29.32	33.50	114.3	95.6	7
b) „ , price-controlled	26.74	26.25	98.2	95.6	8
c) Machine-sewn	22.75	22.50	98.9	97.5	10
Court choes:					
a) price-controlled	24.72	24.25	98.1	100.0	5
b) free	25.82	32.02	116.2 (124.0)	100.0	5
Children's boots	20.25	19.75	97.5	96.2	20
Children's shoes	17.25	17.00	98.6	97.2	20
Slippers	7.53	17.00	97.5 (225.8)	97.8	4
Soling and heeling (c)	8.73	9.09	104.1	104.3	67
Goloshes—rubber boots (c)	5.60	16.50	94.5 (294.6)	94.6	12
All shoe items			*102.0*	*99.6*	*182*

official index was based on special figures obtained from the price control authorities.

Workman's shirt: To September 1947 article No. 10 100; from December 1947 article No. 10 500.

Hat. To December 1948 made of woollen felt; from March 1949 made of hair felt.

Cap. To December 1948 made of carding cloth; from March 1949 made of worsted cloth.

Woollen oversocks. To December 1947 average of No. 11 ½ and No. H.R. 1066; from March 1948 No. BA 1045.

Knitting wool. To December 1948 P.P. Super T or B.B.; from March 1949 Elit, Edit, Elektra, Nornan.

Table 4.3.—*Shoes*

Court shoes, free. To March 1947 black; from June 1947 coloured.

Slippers. To December 1948 made of felt; from March 1949 made of leather.

Goloshes. To December 1948 man's goloshes; from March 1949 rubber boots for ladies.

Table 4.4.—*Miscellaneous items*

Item	Quantity	Average price, Kr.		Index of average prices	Index actually used	Weight, Kr., Dec. 1946
		Dec. 1946	Dec. 1949			
Table	one	132.10	127.14	96.2	96.0	12
Chairs, upholstered	4	142.70	145.08	101.7	96.7	12
Kitchen chair	one	9.28	12.87	138.7	106.5	11
Bed	„	24.82	30.24	121.8	105.2	11
Ceiling lighting	„	9.22	9.74	105.6	99.3	12
Easy chair	„	81.26	79.55	97.9	99.9	11
Curtain (c)	meter	2.22	2.60	117.1	101.6	7
Mattress	one	34.79	35.76	102.8	97.6	14
Towels	dozen	24.73	28.67	115.9	101.1	7
Sheeting	meter	3.00	3.80	126.7	126.8	14
Mat (c)	one	244.00	300.00	123.0	123.0	7
Plates	dozen	11.08	15.92	143.7	114.5	5
Coffee set	„	9.09	14.74	162.2	116.2	5
Tumbler	„	3.94	6.01	152.5	113.6	5
Earthenware jar (c)	one	2.39	2.85	119.2	107.0	5
Table spoons (c)	dozen	22.81	21.15	116.8 (92.7)	98.2	5
Knives (c)	„	22.24	27.20	98.3 (122.3)	96.5	5
Fire-proof dish (c)	one	3.13	3.15	100.6	102.6	5
Bucket (c)	„	2.58	2.75	106.6	107.0	3
Saucepan (c)	„	5.55	5.25	94.6	94.6	3
Coffee pot (c)	„	4.15	3.95	95.2	95.2	3
Casserole (c)	„	5.75	5.60	97.4	97.4	3
Washing board (c)	„	2.91	3.50	120.3	97.6	3
Nails	1000	1.50	1.60	106.7	106.7	2
Hammer (c)	one	1.85	1.85	100.0	95.7	3
Bow saw (c)	„	7.90	7.95	100.6	100.6	3
Sweeping brush (c)	„	3.79	4.27	112.7	105.2	3
Electric iron (c)	„	17.49	16.84	96.3	96.3	3
Vacuum cleaner (c)	„	195.00	215.00	110.3	103.6	3
Sewing machine (c)	„	411.37	450.00	97.4 (109.4)	95.4	2
Electric bulb (c)	„	1.10	0.90	81.8	81.8	3
Torch battery (c)	„	0.75	0.71	94.7	94.7	2
Wall mirror (c)	„	12.35	9.00	95.1 (72.9)	95.1	3
Kitchen paper, crèpe (c)	roll	0.57	0.75	131.6	108.9	3
Oil paint (c)	kg	3.30	3.65	110.6	110.6	2
Toilet soap (c)	one	0.67	0.68	101.5	101.5	25

Table 4.4.—*Miscellaneous items* (cont.)

Item	Quantity	Average price, Kr.		Index of average prices	Index actually used	Weight, Kr., Dec. 1946
		Dec. 1946	Dec. 1949			
Soap powder (c) . .	kg	2.43	1.88	77.4	77.4	33
Soft soap	½ kg	0.44	0.43	97.7	97.7	7
Washing and ironing	—	1.44	1.02	109.2 (70.8)	105.2	4
Laundry	kg	0.83	0.88	106.0	106.0	8
Turkish bath . . .	time	0.98	1.07	109.2	106.0	8
Shaving (c)	„	0.49	0.60	122.4	116.3	3
Hair cutting . . .	„	1.84	2.09	113.6	113.6	14
Tooth paste (c) . .	tube	1.58	1.50	94.9	94.9	9
Razor-blades (c) . .	10	3.00	2.85	95.0	95.0	6
Arch support (c) . .	pair	17.40	16.55	95.1	95.1	3
Medicine (c)	—	.	.	.	93.0	34
Doctor's fee (c) . .	—	.	.	.	100.0	11
Hospital treatment (c)	—	.	.	.	100.0	8
Dental treatment (c)	—	.	.	.	100.0	8
School books (c) . .	—	.	.	.	99.8	13
School fees (c) . . .	—	.	.	.	99.8	17
Newspaper, subscription (c) .	year	23.69	28.41	119.9	121.8	13
„	copy	0.13	0.17	130.8	138.5	14
Books (c)	—	.	.	.	112.9	15
Weeklies (c) . . .	copy	0.31	0.44	141.9	125.2	13
Trade union subscription (c)	—	.	.	.	98.6	77
Club subscriptions (c)	—	.	.	.	137.3	19
Sickness insurance (c)	—	.	.	.	104.9	67
Life and accident insurance (c) . . .	—	.	.	.	106.1	107
Cinema	ticket	1.54	2.01	130.5	129.2	31
Theatre (c)	„	3.29	3.54	107.6	107.6	6
Sporting events . .	„	1.65	1.73	104.8	104.3	10
Wireless license (c) .	year	10.00	10.00	100.0	100.0	10
Toys (c)	—	.	.	.	99.8	3
Gramophone record (c)	one	4.85	4.88	100.6	100.6	4
Wireless set (c) . .	„	265.00	275.00	103.8	103.8	17
Wireless valve (c) .	6	102.00	70.00	68.6	70.0	7

Table 4.4.—*Miscellaneous items* (cont.)

Item	Quantity	Average price, Kr.		Index of average prices	Index actually used	Weight, Kr., Dec. 1946
		Dec. 1946	Dec. 1949			
Bicycle (c)	one	188.50	195.00	100.4 (103.4)	97.5	26
Skis	pair	27.96	28.48	101.9	103.0	10
Tram fare (c) . . .	time	0.20	0.28	140.0	118.5	27
Train fare (c) . . .	„	9.84	11.66	118.5	118.5	33
Bus fare (c)	50 km	0.84	0.97	115.5	109.5	31
Interest on loan (c) .	—	.	.	.	100.0	6
Telephone subscription (c)	—	.	.	.	122.7	15
Inland postage for letter (c)	stamp	0.20	0.20	100.0	100.0	8
Stationary (c) . . .	—	.	.	.	99.5	5
Domestic help . . .	day	7.68	13.60	135.6 (177.1)	136.4	39
Smoking requisites .	—	.	.	.	110.6	72
Spirits and wines (c)	—	.	.	.	117.9	128
Man's wrist watch .	one	95.00	135.00	124.2 (142.1)	96.0	5
Brief case (c) . . .	„	42.00	60.00	92.3 (142.9)	80.0	5
Suit case	„	13.52	13.99	103.5	104.1	5
Other outlay, not specified	—	.	.	.	108.2	87
All miscellaneous items				*110.3*	*108.0*	*1326*

Table 4.4.—*Miscellaneous*

Table spoons. To December 1948 electroplated; from March 1949 made of stainless steel.

Knives. To December 1948 Jernbolagets No. 432 or similar, prices collected by agents; from March 1949 No. 210 P, prices collected centrally.

Sewing machine. To March 1949 average of prices for Husqvarna Klass 12/330 and Singer 201 K 1; from June 1949 average of Husqvarna Klass 12/390 and Singer 201 K 1.

Wall mirror. To September 1947 one size; from December 1947 smaller size; from March 1948 again smaller size.

Washing and ironing. To December 1948 5 starched collars; from June one shirt with attached collar.

Bicycle. To September 1947 average of one welded and one soldered; from December 1947 only one welded.

Domestic help. To December 1948 including board; from March 1949 excluding board.

Man's wrist watch. To June 1949 Tissot militär; from September 1949 Tissot militär, large model.

Brief case. To September 1947 made of horse-hide; from December 1947 made of ox-hide.

Chapter 5.—The Quality Problem and the Indifference Defined Index

The character of the quality problem has been examined in chapter 3 and its numerical significance in chapter 4. This treatment of the problem should be regarded as an introduction to a more thorough analysis of what should be meant by quality and how it should be measured. An approach to the theoretical problem will now be attempted from the standpoint of the indifference defined 'point-to-point' index.

The problem

As usual only one consumer is considered and it is assumed that he maximizes his utility function. The consumer is confronted with a number of goods which is practically infinite. It has, of course, long been recognized that every different variety must be considered as a separate commodity. The following quotation makes it clear that it is necessary to describe the physical characteristics of the articles rather fully:

"The economic universe is not made up of hats, autos, and spinach. It is made up of felt hats, straw hats, dress hats, sport hats, blue hats, black hats, aeroplane helmets, nightcaps, and, on the edge, triangular scarves; and Plymouths, Fords, Chevrolets, roadsters, coupes, and convertibles; and plain spinach, fancy spinach, spinach in cans, and Birds Eye frozen spinach."[1]

That it is necessary also to consider other characteristics than the physical ones has been stressed by Chamberlin, from whom the following passage may be quoted:

"Differentiation may be based upon certain characteristics of the product itself, such as exclusive patented features; trade-marks; trade names;

[1] *Norris* (1941), p. 77.

peculiarities of the package or container, if any; or singularity in quality, design, color, or style. It may also exist with respect to the conditions surrounding its sale. In retail trade, to take only one instance, these conditions include such factors as the convenience of the seller's location, the general tone or character of his establishment, his way of doing business, his reputation for fair dealing, courtesy, efficiency, and all the personal links which attach his customers either to himself or to those employed by him. In so far as these and other intangible factors vary from seller to seller, the 'product' in each case is different, for buyers take them into account, more or less, and may be regarded as purchasing them along with the commodity itself. When these two aspects of differentiation are held in mind, it is evident that virtually all products are differentiated, at least slightly, and that over a wide range of economic activity differentiation is of considerable importance."[2]

Any one single consumer buys only a limited number and as a matter of fact only comparatively few articles of the totality of goods. And, as pointed out by Hicks, "if a consumer does not buy *any* of a particular good, then his marginal valuation of that good must be *less* than the market price; it may be only a little less, but it may well be a great deal less."[3]

As pointed out by Norris "practically all goods occur in the consumption pattern of individuals in clusters of competing substitutes."[4] Norris further emphasizes that "from the point of view of each consumer, the emphasis in cluster formation is upon the services the goods are able to render."[5] Picking out any single piece of goods, i.e. a completely specified article, one will find that it is surrounded by a number of other articles (other makes, brands etc.), which are close substitutes to it. There are hardly any exceptions to this rule. Only a few articles could be mentioned for which there are no quite close substitutes (milk, telephone service, postage for a letter, wireless licence).

The most frequent relation between one article and other articles outside the cluster of its substitutes will also be one of substitutability, which will be weaker, the less kindred the items are. One will also find that some related items are complementary (electric current-

[2] *Chamberlin* (1948), pp. 56–57.
[3] *Hicks* (1948), p. 163.
[4] *Norris* (1941), p. 76.
[5] *Norris* (1941), p. 78.

light bulbs, cars-tyres, tobacco-pipes etc.), but such relations will as a whole be rare.

When Hicks states that "any particular good will have a little knot of other goods round it that are complementary with it; but its most probable relation with any other good taken at random will be one of (doubtless mild) substitutability,"[6] he thus overlooks the close substitutes in the immediate neighbourhood of almost all goods. The relation between goods which are close substitutes is, however, a most important one.

As pointed out by Hicks a group of goods can be treated as a single commodity so long as relative prices are unchanged.[7] However, it is just the validity of the assumption that relative prices of substitute goods remain constant that is challenged in the present analysis.

We now come to the index problems. The first thing which we wish to obtain is an indifference defined index for a group of goods which are substitutes. All other goods are thus left out of consideration. To start with the case with only two goods, *a* and *b*, is treated.

Let us now form the definition that two goods are substitutes if a certain quantity of the one article can always be substituted by a certain, usually different, quantity of the other article. This means that the indifference curves for the two goods must meet both axes. Otherwise the curves may have any form, i.e. they may be convex, straight lines or concave to the origin. These three cases will now be treated separately.

The indifference curves are convex

We consider the case with convex indifference curves when both goods, *a* and *b*, are for sale in situation 0, but when the consumer prefers to buy article *a* only. This position is illustrated in fig. 5.1, where the budget line *AB* meets one of the indifference curves in *A*.

If now in situation 1 the price of *a* is unchanged but the price of *b* is lower than in situation 0, the new budget line *AB'* may intersect the original indifference curve. In such a case the price quotient for *a* (= 1) will not give the correct price index for the two goods in common. If, supposing the consumer to have his original income, in-

[6] *Hicks* (1946), pp. 47–48.
[7] *Hicks* (1946), p. 33 and p. 312. Cf. *Samuelson* (1947), p. 136.

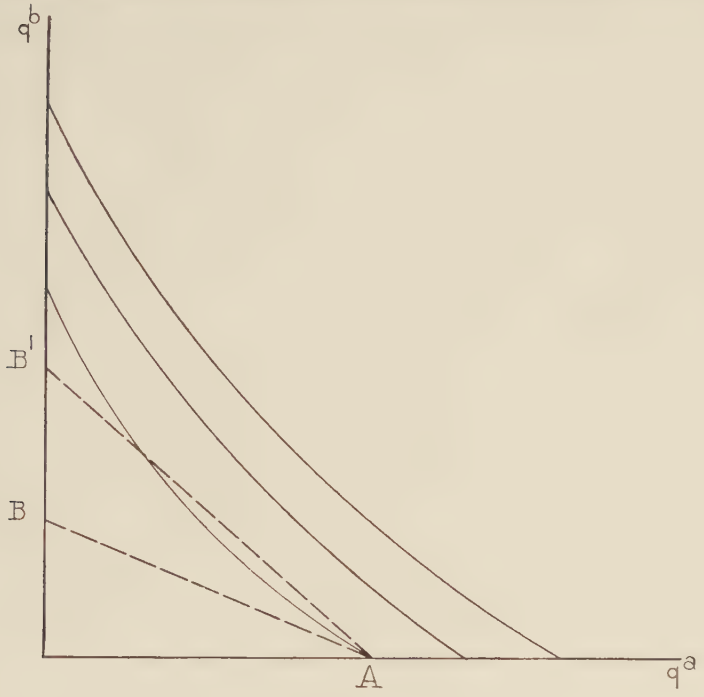

Fig. 5.1

come here meaning money available for the purchase of the two goods, he may, by partly substituting *a* for *b*, attain a higher indifference level than the original one, and the true index for the two goods will be less than $\frac{p_1^a}{p_0^a} = 1$, by an amount depending upon the convexity of the curve and the magnitude of the price difference for *b*.

If the price of *b* in situation 1 is very much lower than in situation 0 (the price of *a* being the same), the situation illustrated by fig. 5.2, may arise. By completely substituting *a* by *b* (the new budget line, *AB''*, as against the original one, *AB*), the consumer may attain a much higher indifference level than the original one. The correct index is obtained, if the income corresponding to the line *A'B'*, is compared with the original income, corresponding to *AB*.

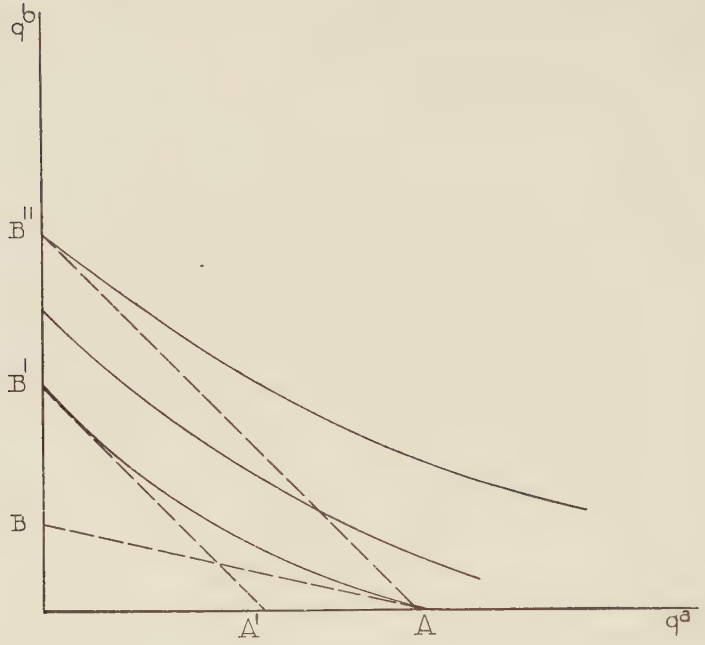

Fig. 5.2

The indifference curves are straight lines

The more alike the two goods are, the less convex are the curves. If the goods are 'perfect substitutes', the indifference curves reduce to straight lines with gradients equal to —1. Fisher, who was the first to point this out, gives as an example coal of the same quality coming from different pits.[8] It is a matter of indifference to the consumer if he receives a certain quantity of the one sort or the same quantity of the other or any combination of the quantities lying on one of the indifference lines in fig. 5.3.

As pointed out by Wold[9], if the indifference curves are straight lines, only their ends can represent optimal budgets. If the prices of (practically) identical articles are different, the consumer will prefer to buy that article only, for which the price per unit is lowest (the

[8] *Fisher* (1892), p. 71; cf. *Wold* (1943), p. 233.
[9] *Wold* (1943), p. 233.

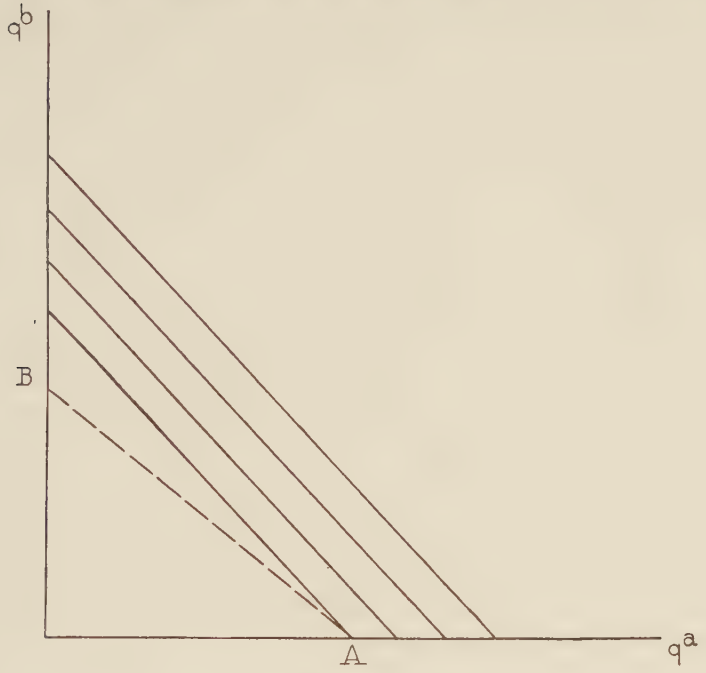

Fig. 5.3

budget line *AB* in fig. 5.3). If both prices and qualities are identical, the budget line and the indifference line will coincide, and the consumer's choice will be unstable.

Indifference curves which are straight lines may also be obtained for goods which are not identical, viz. if a certain quantity of one article corresponds to a different quantity of another. If, now, the relation between the prices corresponds to the quality relation between the articles, the budget line and the indifference line will coincide and the situation will again be unstable.

One can for instance imagine two kinds of coal, the one having twice the heat value per kg of the other.[1] If the first-mentioned variety costs twice as much as the other, the consumer may buy either sort. The above-mentioned case, where both qualities and

[1] We disregard other differences, e.g. that the coal with a higher heat value may be easier to handle, requires less space for storage etc.

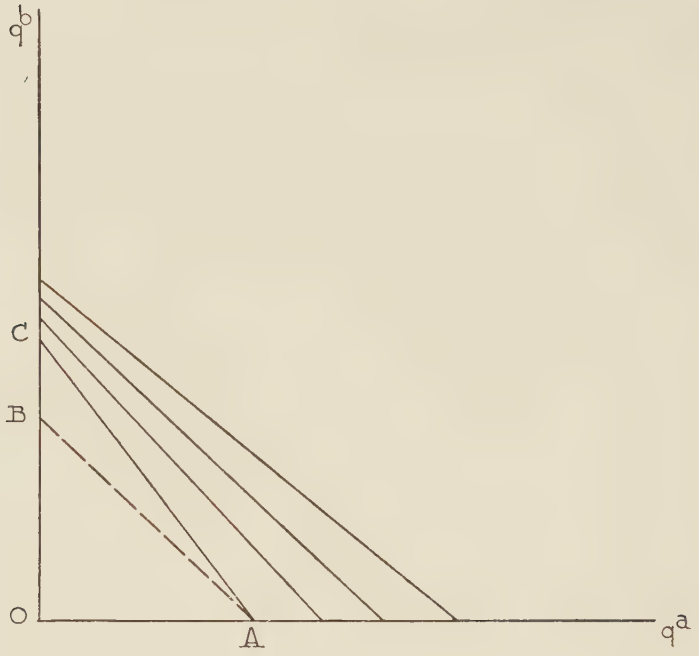

Fig. 5.4

prices are identical, is thus seen to be a special case of this more general situation.

As soon as the one article is *more expensive in relation to its quality* than the other, the budget choice will again be stable, and the diagram will look like fig. 5.4. The indifference lines may of course now have any slope and need not be parallel.

The indifference curves are concave

This situation is illustrated in fig. 5.5. The budget line may now connect the ends of an indifference curve, which means that it is a matter of indifference to the consumer whether he buys only article *a* or only article *b*, whereas any combination of the goods, represented by the budget line (*AB*), will bring him down to a lower indifference level. In general a certain budget line will not meet more than one end of an indifference curve (*A'B*), this end (*B*) representing

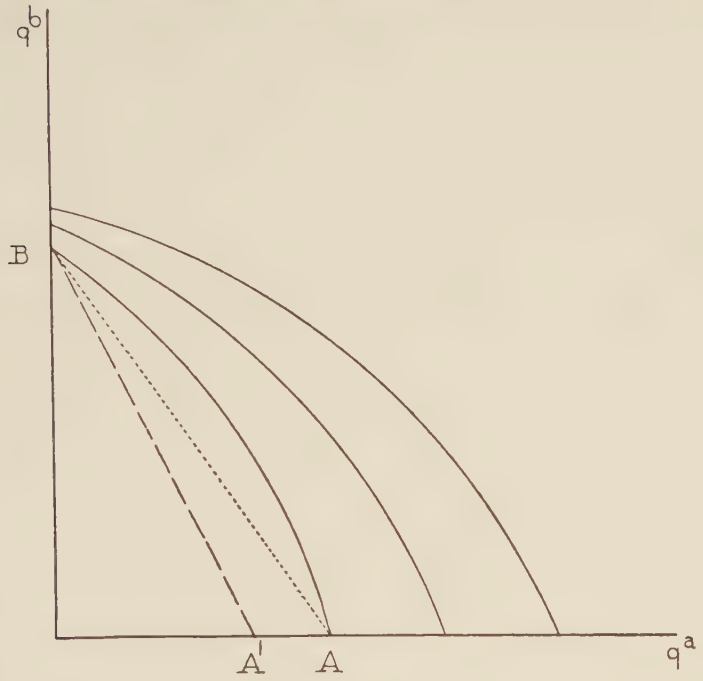

Fig. 5.5

the consumption which brings the consumer to the highest indifference level.

In the literature it is generally assumed that concave indifference curves are of a purely theoretical interest and rarely occur in practice. Hicks describes this as "a very queer case"[2] and in his comments on this statement Norris says that "I doubt if Hicks realized *how* queer."[3] At the same time, however, Norris emphasizes that "indifference curves of substitutes *may* be concave to the origin in shape."[4]

Wold has shown that when the indifference curves are convex, the budget choice is governed by the 'principle of compromise,' whereas concave curves correspond to a budget choice governed by

[2] *Hicks* (1946), p. 16.
[3] *Norris* (1941), p. 36.
[4] *Norris* (1941), p. 31.

the 'principle of either-or.'[5] He gives examples of both cases but concerning the latter he says that "it seems difficult to give a convincing example of a budget choice governed by the principle of either-or."[6]

When substitutes are considered and all other goods are left out of consideration, this situation may, however, very well occur. Even if the prices of two varieties are such that it will be a matter of indifference to the consumer whether he buys the one or the other, he may well prefer to choose one of them rather than a combination of both. To buy two or more kinds of, for instance, cigarettes, coal, razor-blades, soap, etc. will often be unattractive or inconvenient to the consumer who will prefer to keep to a certain kind (brand, make etc.).

What is quality?

The indifference diagram refers to one specific individual in a certain situation. The distances from the origin, at which the axes are touched by a certain indifference curve—which may be convex, a straight line, or concave—indicate how the consumer judges the quality relation of the two substitute goods. This valuation is strictly limited to one consumer with a given income for the purchase of the two goods. Quality as defined here has nothing to do with any 'objective' valuation of the goods, e.g. in the form of a comparison of caloric contents of foods, durability of clothes etc. The indifference diagram may look quite different for another consumer or for the same consumer in a different situation.

The consumer may clearly value the two goods in a different way, when the income varies. When the indifference curves are straight lines, this means that they are not parallel. If the income-consumption curve is considered, the consumer will, when the income increases, buy larger and larger quantities of the one variety, until he at last goes over to the other variety.[7] If the indifference curves are concave or straight lines, there will be a certain income at which consumption will be unstable; and if they are convex, there will be a certain income range, within which he will distribute his purchases

[5] *Wold* (1943), pp. 232–4.
[6] *Wold* (1943), p. 234.
[7] This has been pointed out by Norris; see *Norris* (1941), p. 36.

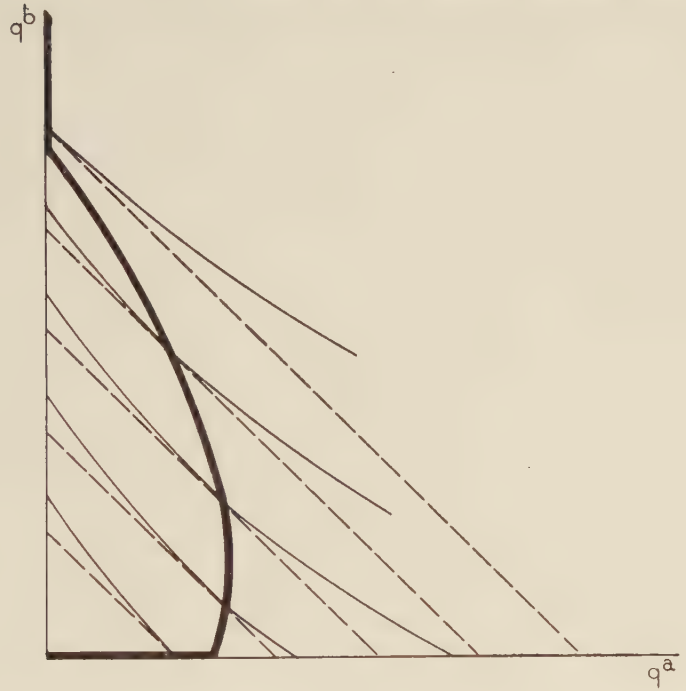

Fig. 5.6

on both goods. This latter case is visualized in fig. 5.6, where the heavy line signifies the income-consumption curve.

The problem which is treated here refers to the computation of an index indicating the common price development for two goods, which are so alike that they may be treated as different varieties of the same article. To start with we will ignore the case when the indifference curves are convex, which means that we assume that the consumer always buys only one variety, and never a mixture of both. From our present point of view it does not matter if the indifference curves are straight lines or concave. In both cases the consumer will buy only the one variety, except in the limiting case when his choice will be unstable. This case is ignored.

Little has stated that convexity (by him in the following quotation called concavity) of the indifference curves must be assumed if all points in the diagram can conceivably represent some market situa-

tion. But every single consumer need of course only buy of one article, and all consumers consequently be distributed along both axes. Little further on says that "If, for instance, the curves were anywhere straight lines, then in some price-income situation there would have been no determinate solution. In fact, there would have been actual indifference. But indifference, it was said, was never noticed; people always made definite choices. Therefore, indifference curves must be assumed to be everywhere concave"[8] (= convex). This does not apply to the case treated here; the indeterminate position may be looked upon as a limiting position which is never exactly fulfilled.

The factor g

Let us now return to the index problems as is the case when both *a* and *b* are obtainable in situation 0, and the consumer buys *a*, this article having disappeared from the market in situation 1 when article *b* is still obtainable at an unchanged price. In accordance with the preceding paragraph the indifference curves between *a* and *b* are assumed to be straight lines or concave.

Assume that the quantity consumed in situation 0 ($= q_0^a$) corresponds to OA (fig. 5.4). Then the quantity of *b*, which corresponds to the same indifference level, is represented by OC. Denote this quantity of *b* by $\frac{1}{g} q_0^a$. Geometrically $-\frac{1}{g}$ is evidently the gradient for the indifference line, or for the straight line connecting the ends of the indifference curve if it is concave.

If a quantity $\frac{q_0^a}{g}$ of *b* is required to bring the consumer to the same indifference level as before, when he consumed q_0^a, the index will be

$$I_{01} = \frac{p_1^b \cdot \frac{q_0^a}{g}}{p_0^a \cdot q_0^a} = \frac{1}{g} \cdot \frac{p_1^b}{p_0^a} \tag{5.1}$$

We have thus arrived at the same fundamental formula as the one used in chapter 3 (designated 3.1). It should be noted that, according to the argument earlier in the chapter, *g* in formula (5.1) may

[8] *Little* (1949), p. 97.

have a different value for another consumer or for the same consumer in another situation.

In this case one must have $g < \frac{p_1^b}{p_0^a}$ and consequently $I_{01} > 1$. Otherwise the consumer would not have preferred a in situation 0. A disappearance of a variety consumed in situation 0 thus indicates a price increase, provided that it is substituted by another variety which was present but not consumed in situation 0 and which has not altered its price.

In situation 0 the outlay is $p_0^a q_0^a$. If the price of b had been lower than p_0^b, the indifference level which is represented by this outlay could have been attained by the consumer, if he bought b. If the limiting case occurs at the price $\overline{p}^b$, then one must have

$$\overline{p}^b \cdot \frac{q_0^a}{g} = p_0^a \cdot q_0^a \text{ or } g = \frac{\overline{p}^b}{p_0^a}$$

The factor g, then, is the quotient between the price of b, less than p_0^b, at which the consumer would have been indifferent to consuming a or b, and the actual price of a ($= p_0^a$). In practice it is, of course, difficult to establish this price $\overline{p}^b$, but in principle such a price must exist and be uniquely determined.[9] If this price is known, then the magnitude of g is also known, and it is possible to use formula (5.1) for the computation of the index.

For the determination of g we have asked; what price of b lower than p_0^b would lead to an indifferent choice? We may not ask instead: what price on a, higher than p_0^a would make the choice indifferent? This question refers to a different indifference level, and the answer will be the same only if the indifference lines are parallel, which they need not be (cf. fig. 5.4).

It may be necessary to use formula (5.1) even when both a and b are obtainable in both situations 0 and 1. Let us assume that the situations are identical, with the exception that the price p_1^b is so much

[9] Here as in other connections it must be kept in mind that the indifference theory is only a kind of working hypothesis, which has turned out to be a great help in the treatment, among other things, of index problems. In practice the consumers never behave so rationally and consciously as the theory presupposes. For a criticism of the indifference theory from such points of view, see *Norris* (1941), especially chapter III, and *Little* (1950). Cf. also chapter 6.

lower than p_0^b that $p_1^b < g p_0^a$. The consumer who in situation o bought only *a*, can then in situation 1 attain a higher indifference level than the original one, if his income is unchanged. The correct index is obtained through formula (5.1), and one will have

$$\frac{p_1^b}{p_0^b} < I_{01} < 1$$

A similar situation occurs if the price of *a* rises, so that a substitution of *a* by *b* becomes advantageous. The price quotient $\frac{p_1^a}{p_0^a}$ does not then give a correct result. Formula (5.1) should again be used, and the factor *g* be determined in the way indicated above.

The volume index should also be considered. As the actual consumption in situation 1—q_1^b—is equivalent to a quantity $g \cdot q_1^b$ of *a*, the volume index is obtained by a comparison between this quantity and the original quantity q_0^a, i.e. one obtains:

$$Q_{01} = g \frac{q_1^b}{q_0^a} \tag{5.2}$$

The product of (5.1) and (5.2) gives the value ratio, i.e. the relation (1.13) is fulfilled.

A new article in the market

Let us assume that in situation o only article *a* is available, but that in situation 1 article *b* has been added to the market and is bought by the consumer (article *a* still for sale at an unchanged price). The quotient $\frac{p_1^a}{p_0^a} = 1$ may, of course, now again give an incorrect result.

The factor *g* should again be determined from an indifference diagram referring to situation o under the assumption that *b* had already been for sale then; see fig. 5.7. If *b* had been present in situation o, the consumer would have preferred this article. *g* is obtained by a comparison between the price $\overline{p}^b$—higher than p_1^b—which would have made the consumer indifferent in situation o, and the price p_0^a. This agrees with Hicks' argument.[1]

[1] *Hicks* (1940), p. 114.

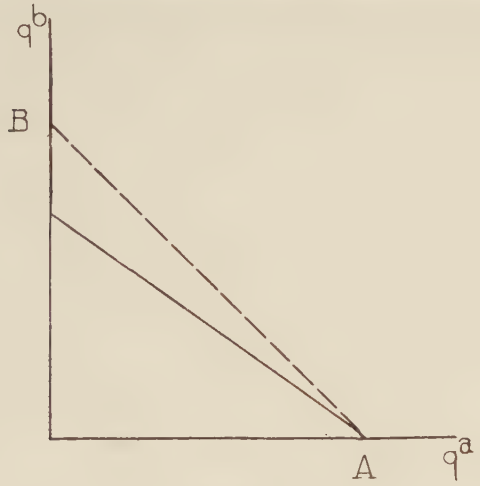

Fig. 5.7

If a transition from *a* to *b* has been advantageous, one must have $g > \frac{p_1^b}{p_0^a}$ and consequently from formula (5.1) $I_{01} < 1$. This means that if a new article has entered the market in situation 1 at a price which is so low that the consumer, according to the indifference diagram and the indifference level in situation 0, has found the transition advantageous, the change should be interpreted as a decreased price.

If in situation 0 only article *a* and in situation 1 only article *b* are obtainable, the argument will follow similar lines. Formula (5.1) should be used, and in order to obtain *g* one must ask what price of *b* would have made the choice indifferent in situation 0, provided *b* had already then been available. In this case one may obtain $I_{01} \lessgtr 1$.

It may seem quite unrealistic to refer the valuation of a new article to a situation in which it is not obtainable (unless the comparison is geographical or the article had been available earlier than in situation 0). How would nylon stockings have been valued in 1939 or a 1952 model car in 1948? This problem is discussed by Little who, among other things, says that "formally, a new good cannot be introduced into an old behaviour-line map.———The best we can do

is to imagine that the consumer knew all about the new good all the time, and that it had a definite place on his scale of likes and dislikes, and that he was simply prevented from buying it, because it had, let us say, an infinite price. This is, of course, unsatisfactory."[2]

This is a serious difficulty. It is, however, an inevitable consequence of the definition of the problem as given here. According to the approach in this chapter it is necessary to value all goods with reference to the situation o, however unrealistic this may seem in the case of goods which were then not available. The nature of this difficulty will be clarified by the discussion in chapter 6.

The case with two goods, *a* and *b*, having thus been treated, some discussion should be given of the case when there are more than two goods within one group of substitutes. But as long as we restrict ourselves to a single consumer, no extra difficulties arise. If all the goods are substitutes, any single consumer will never buy more than one article at a time (except in limiting cases which may be neglected). The transition from one article to any other article within the cluster of substitutes may be treated in the manner described above. No other articles are relevant.

The index for all goods

Naturally we are not only interested in separate indexes for groups of substitutes but also—and primarily—in an index for all goods. In order to make the transition from groups of substitutes to an aggregate index for all goods, let us consider an indifference diagram in which one axis represents one group of substitutes, i.e. quantities of the specific article, *a*, among the substitutes which are consumed, and the other axis represents all other articles which are consumed, *r*.[3]

In a diagram of this type the curves must be convex to the axes along their whole length; see fig. 5.8. No kinks are possible[4] and these curves may not be concave or straight lines, as this would invalidate the assumption that both *a* and other goods are consumed.

It is further seen that the curves must start from a point on the q^r-axis and that they may not touch the q^a-axis, (there is no single

[2] *Little* (1950), pp. 166–7.
[3] Cf. *Hicks* (1946), p. 33, and *Norris* (1941), p. 27.
[4] Cf. *Hicks* (1946), p. 23.

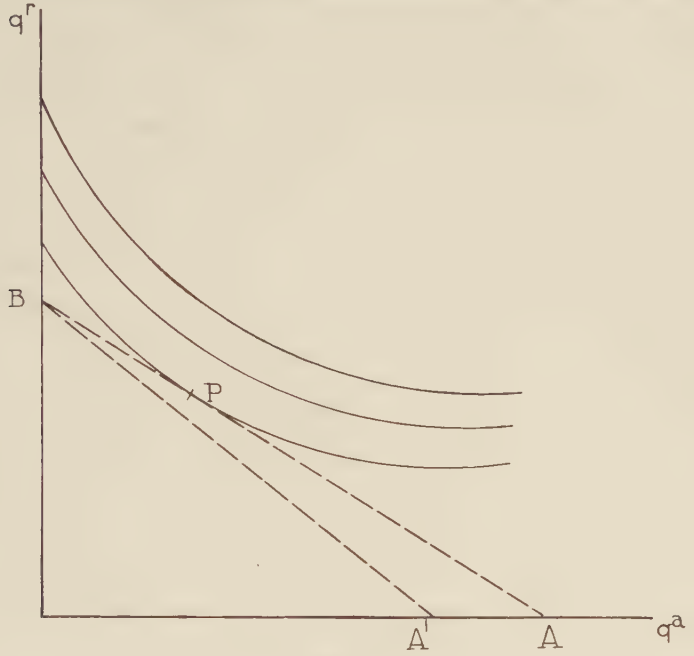

Fig. 5.8

article *a*, which the consumer is not prepared to give up completely, and no single article is of such great importance to the consumer that he can be satisfied by getting that article alone and nothing of other goods). Every curve will have an asymptote parallel to the q^a-axis, and the q^r-value of the asymptote will correspond to the position in which the consumer gets an unlimited quantity of *a*.

We consider the case when there has been a price increase for *a* but when all other prices are unchanged. The original income gives the budget line *A'B*. This may result in an increased quantity of *a* (Giffen's paradox), but in most cases the quantity of *a* will be reduced. If the consumer is Laspeyres-compensated (the budget line through *P* parallel with *A'B*), the quantity of *a* must be reduced, whereas the expenditure for other goods must have increased. This follows from the convexity of the curve.

As demonstrated in chapter 1, an exact solution of this index problem presupposes knowledge of the form of the curve. In practice

such knowledge is never available, at any rate not for more than a few items. One is forced to rely upon assumptions and to estimate in general terms the magnitude of the errors which are committed if the one procedure or the other is chosen. In the first hand the effect of using Laspeyres' formula should be considered.

The difference between the 'true' index and the result obtained from Laspeyres' formula, depends upon the curvature of the indifference curve at the point P. In fig. 5.9 $\varphi_0(r,a)$ and $\varphi_0(r,b)$ are the indifference curves between a and other goods, and b and other goods respectively. Both curves have the budget line AB as tangent at point P.

After a certain price increase for a—or for b—Laspeyres' formula is represented by the budget line $A'B'$. The difference between this index and the 'true' index is seen to be smaller the greater the curvature of the indifference curve. The 'true' index for article b is thus represented by $A''B''$, whereas for article a with a less marked curvature this index is obtained from the budget line $A'''B'''$.

Returning now to fig. 5.8, it is thus obvious that the relation between any single commodity, a, and all other goods, r, will be different for different articles. We consider the case when there has been a price increase on the article, a, and the consumer has been Laspeyres-compensated. Depending upon the nature of the article a, i.e. the form of the indifference curve, its consumption will have been reduced in different degrees. Consumption of other articles may also have been affected in a different way: it may thus have increased a little for all other goods or they may have developed differently. For goods which are complementary to the article a, consumption may thus have been reduced and for goods which are supplementary to a, consumption may have increased considerably.

If there are no specific substitutes for a certain article among 'other goods', and if the price change for the article in question has been moderate, it must be legitimate to use Laspeyres' formula, i.e. to assume that the consumer is only compensated for a price change on a if he receives an income which makes it possible to maintain unchanged the quantities of both a and other goods. A glance at tables 4.1—4.4 will suffice to show that there are quite a number of goods, or rather clusters of goods, for which the possibilities of such specific substitu-

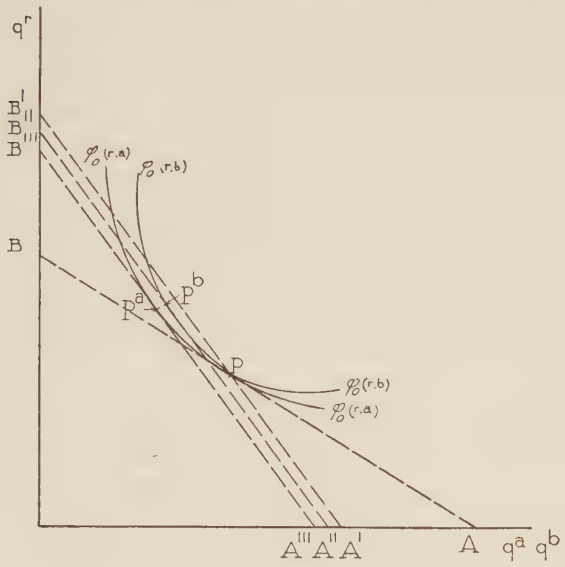

Fig. 5.9

tions must be considered as rather remote. One can possibly substitute spoons for forks but not vice versa, and neither of them can be substituted by anything else. One can sit upon a bed, but scarcely sleep on a chair.

The more specific substitution possibilities for the article *a* among 'other goods', the more misleading it will be to use Laspeyres' formula when the price of *a* has changed. This is seen to be the case with food: every separate food item can be replaced by other kinds of foodstuffs as different qualities of the same article: food. In the same way one kind of fuel can be substituted by other kinds, cinema tickets can be replaced by other kinds of leisure time activities, e.g. theatre tickets, tickets for sporting events, books etc.

In this section the quality problem is not considered, i.e. only articles which are really consumed in situation o are taken into account. When this is done, the conclusion from the above will be that the bias introduced by the use of Laspeyres' formula will on the whole be small for the clothing and miscellaneous items groups, whereas the bias may be greater for food and fuel items etc.

Sub-indexes for groups of items

According to the theory for an indifference defined index the income must refer to the whole income of the consumer, and when computing the change in income required by the consumer to remain on the same indifference level, when the price of one article is altered, the hypothetical (not the actual) change in the consumed quantities of *all* other goods must be taken into account. A sub-index for a group of items, e.g. a food index, a shoe index, a furniture index etc., is consequently not conceivable, as there is nothing like 'income for food' etc.

In order to define sub-indexes for groups of items, it is necessary to assume that these groups are independent of one another.[5] This assumption is evidently implicit when Laspeyres' formula is used. The price changes for every single item then affect the total index in a manner without affecting other items. When Laspeyres' formula is used, sub-indexes for groups of items may well be and in practice very often are computed.[6]

It may be noted that if a group of goods is independent of other goods, Giffen's paradox may occur for an article which plays a small part in consumption as a whole. Such a situation will arise when the article is inferior, and when it plays a great part among the group of goods which are independent of all other goods.

So far only goods which are actually consumed have been taken into consideration. We now come to the case when articles $r+a$ are consumed in situation 0 but when in situation 1 variety a has disappeared and been substituted by another variety b, available but not consumed in situation 0.

In order to study this case, let us consider the position of the consumer with respect to the article b in situation 0. In fig. 5.10 the curve $\varphi_0(r,a)$ is the indifference curve between article a and all

[5] If the groups are designated A, B, C etc. this means that the utility function must have the form

$$U = \alpha(A) + \beta(B) + \gamma(C) + \ldots\ldots$$

[6] The resolution, adopted by the Second International Conference of Labour Statisticians as a matter of fact contains a recommendation that "separate index numbers should be published for each of the groups" (i.e. food, clothing, heating and lighting, rent, and miscellaneous). See *Conference* (1925), p. 69.

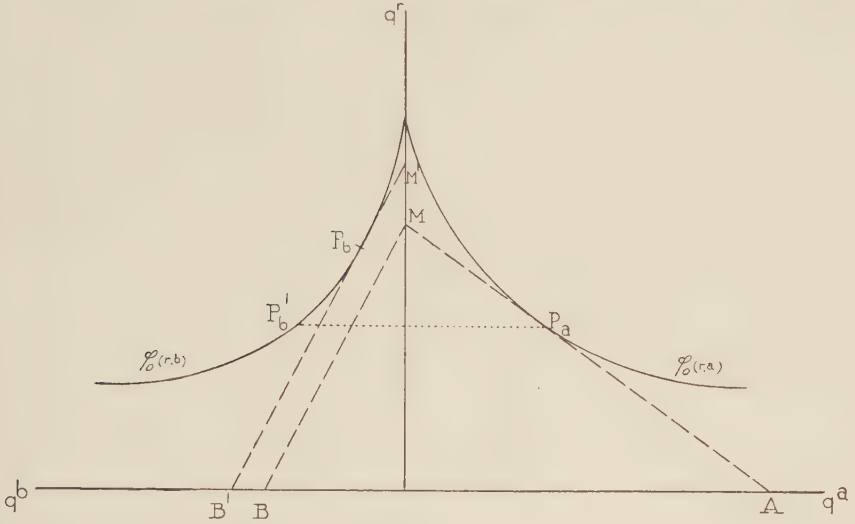

Fig. 5.10

other goods, and $\varphi_0(r,b)$ the curve between article b and all other goods, *corresponding to the same indifference level.*

The point P_a represents the actual consumption in situation 0, the budget line AM corresponding to the quantities of a and of other goods (r) which may be bought for the given income. The budget line BM represents the quantities of b and of other goods which can be bought for the same income. This budget line cannot intersect the curve $\varphi_0(r,b)$; if this were the case the consumer would of course prefer $r+b$ to $r+a$, which is against hypothesis. The limiting case when AM is a tangent to $\varphi_0(r,a)$ and, simultaneously, BM to $\varphi_0(r,b)$ and the consumer consequently indifferent whether he buys $r+a$ or $r+b$ is neglected.

The point P_b' in fig. 5.10 represents the quantities of b necessary, if the consumer shall remain on the same indifference level as the original one when buying r and b, without altering the quantities of r. The income corresponding to P_b' is obtained in the way described above from the indifference diagram, fig. 5.3, where all other goods are left out of consideration. This point is not, however, a point of optimum satisfaction. If the consumer receives an income correspond-

ing to the budget line through P_b' (parallel to BM), he will move to a higher indifference curve. The point of optimum satisfaction is obtained as the point of contact between $\varphi_0(r,b)$ and the budget line $B'M'$ parallel to BM. This point, P_b, gives the minimum income necessary for the consumer to attain the same indifference level when buying $r+b$ as when buying $r+a$. It is thus seen that when goods $r+a$ are consumed in situation 0 and when in situation 1 article a has disappeared and been substituted by article b, which was obtainable but not consumed in situation 0, and when everything else is unchanged, the price index with reference to the indifference level in situation 0 equals the ratio between the incomes referring to points P_b and P_a.

The question is now, under what conditions it may be considered legitimate to base an index comparison upon the incomes corresponding to P_b' and P_a instead of upon the incomes corresponding to P_b and P_a. In other words, when may one neglect the effect of other goods (r) if a quality transfer from variety a to variety b has taken place? This is what was done in the beginning of this chapter when figs. 5.1–5.7 were considered.

Here it seems impossible to set up more than vague rules. Under normal periods when most quality changes are small and gradual it must as a rule be legitimate to compute the effect of a transfer from one variety to another without taking other articles into account. If, for instance, woollen socks are substituted by nylon socks, it is not necessary to consider other articles.

However, in some cases and especially during periods of war or other disturbances, the quality changes may have a different character, and it may be misleading to treat the substitution of a by b without taking other goods into account. If b is of a much lower quality than a, one may, for instance, come to the conclusion that in situation 0 the consumer would not have been willing to substitute a by b, even if it had cost nothing at all. In that case, if variety a has disappeared in situation 1 it is absurd to base an index computation for a and b on fig. 5.3. As an example one may take the substitution during the war of coffee by coffee-substitute, or of wollen suits by suits made of staple fibre (cellull).

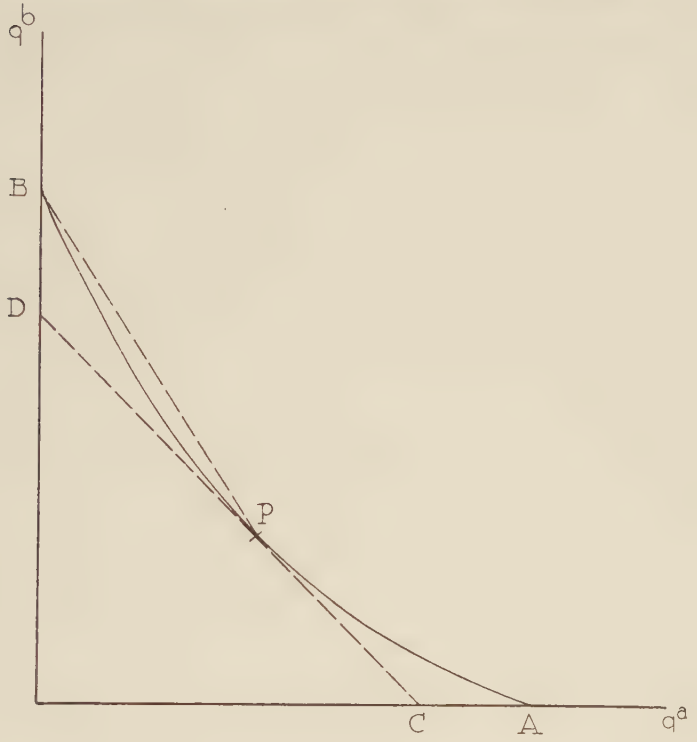

Fig. 5.11

The index problem when the indifference curves between substitutes are convex

So far it has been assumed that the indifference curves between the two substitutes are straight lines or concave. This need not be the case; for many substitute articles it must be assumed that the indifference curves are convex, though only slightly so.

We consider the case when the consumer buys both articles; see fig. 5.11, where the consumer is situated at *P*. Our problem is to find the indifference defined index for the two articles when one of them—say *a*—has disappeared from the market and other articles are not taken into account. It is easily seen that this index is obtained if the income corresponding to the point *B* is compared with the given income corresponding to the budget line *CD*. This quotient must be

larger than unity, i.e. once again we arrive at the conclusion that the disappearance from the market of an article actually consumed in situation o must be interpreted as a price increase, provided that prices remain unchanged.

In order to obtain the income corresponding to the point *B* we thus ask: what minimum income is required in order that the consumer shall reach the given indifference curve while consuming article *b* only? To answer this question one must have some knowledge of the form of the indifference curve through *P*. The greater the curvature, the higher the required income will be, as compared with the given income.

At the point *P* in the diagram the factor *g* equals the marginal rate of substitution[7], i.e. it gives the quotient between the infinitesimal quantities of *b* required in order to compensate the consumer for infinitesimal losses of *a*. Consequently $-\frac{1}{g}$ is the gradient of the budget line. Formula (5.1) gives an index equal to unity.

However, if variety *a* has disappeared suddenly, the consumer is not compensated by an index equal to unity, as an unchanged income does not make it possible for him to attain a higher indifference level than that represented by the point *D*. In order to be compensated for the loss of variety *a* he must receive an income which makes it possible for him to attain point *B*. If, now, in this case $-\frac{1}{g}$ denotes the gradient of the line *PB*, the quantity of *b* required in order to compensate for the loss of *a* may be expressed in terms of *a*, i.e. whereas the actual expenditure in situation o was $p^a q_0^a + p^b q_0^b$, the expenditure required in situation 1 in order to give the same indifference level will be $p^b\left(\frac{1}{g} q_0^a + q_0^b\right)$. The index formula consequently becomes

$$I_{01} = \frac{p^b \left(\frac{1}{g} q_0^a + q_0^b\right)}{p^a q_0^a + p^b q_0^b} \tag{5.3}$$

[7] Cf. *Hicks* (1946), p. 20.

The flatter the indifference curves, the less will the result obtained from formula (5.3) deviate from unity. In the limiting case, when the indifference curve is a straight line, one will have $g = \frac{p^b}{p^a}$ and consequently $I_{01} = 1$.

It goes without saying that if the indifference curve is a straight line and the consumer is situated on the curve—not at one of its end points—it is a matter of indifference to him if one of the two articles disappear. This is again the limiting case which has been neglected above.

Not only the two varieties *a* and *b* but also other goods, *r*, should be taken into account. It is seen that when the indifference curve between *a* and *b* is convex, a simple two-dimensional representation corresponding to fig. 5.10 is not feasible. Three dimensions are required if it shall be possible to demonstrate the position of the consumer when he buys both *a*, *b*, and *r*.

We will not enter further into an analysis of this case. Suffice it to say that, just as in the above-mentioned case when the indifference curve between *a* and *b* is a straight line, it is not possible to lay down any general rules for the computation of the index when *a* or *b* has disappeared. Depending upon the character of the disappearing article, i.e. its importance and its relation to its nearest substitutes and to other goods in general, it may or it may not be an allowable approximation to treat *a* and *b* as independent of other goods.

Conclusions

The preceding theoretical arguments may lead to the conclusion that when more important quality changes have taken place a price index comparison should be based upon a detailed analysis of the relationship between the disappearing or the new article and all other goods, not only the nearest substitutes. In practice, however, such an analysis is not feasible; even if the required data were available, which they never are, it is necessary to base the computation of regular index series on a simpler and quicker procedure. For normal periods the following course of action seems to be recommendable.

The universe of goods is divided into a limited number of groups, all articles belonging to one such group being treated as substitutes for one another. Such a classification will, of course, meet certain difficulties.[8] If there are three goods, *A*, *B* and *C*, it may for instance be possible to classify both *A* and *B*, and *B* and *C* as substitutes. However, from this does not necessarily follow that *A* and *C* are substitutes. These difficulties do not seem too serious, however, and will here be ignored.

From tables 4.1–4.4 it is seen that the number of groups of substitutes should in principle be fairly high. This is especially the case for the 'miscellaneous items.' From table 4.4 it can easily be seen that the number of groups is much higher than the items specified in that table. Not only skis should be mentioned, for instance, but other kinds of sporting articles. In practice, however, the number of groups in tables 4.1–4.4 should at any rate be sufficient.

During normal periods, when both price changes and quality changes are fairly small under a moderate time interval, it must be legitimate to treat all the items specified in this way as distinct one from the other. For each item a separate price index should be computed, and when quality changes occur formula (5.1) should be employed. These separate indexes should be united into an all items index in the way discussed in chapter 1.

However, during periods of war or of other similar disturbances the quality changes may be so important that it becomes absurd to treat all the groups of substitutes as independent of one another. This is especially so if consumption within a group is limited by rationing, or if all the articles within a group disappear from the market.

Rationing and the index

This problem has been treated by several authors.[9] The main line of thought is that one should for the index use the 'virtual price system' which would give the quantities actually used according to the rationing scheme.[1] In essence this problem is the same as the one treated above. A satisfactory solution can only be found if one knows

[8] The difficulties are discussed in *Norris* (1941), p. 76 ff.
[9] See *Rothbarth* (1940–41), *Kaldor* (1940–41), *Nicholson* (1942–43).
[1] *Rothbarth* (1940–41), p. 100.

the demand pattern of the rationed item. For a regular index series rationing must, however, be allowed for in a simpler fashion.

In some cases it may be assumed that the reduction in consumption of the rationed commodity is compensated for by increased consumption of another commodity, e.g. coffee by coffee-substitute. The substitution may then be openly demonstrated in the index calculations. However, if formula (5.1) shall be used, how shall g be determined? A strict adherence to the principles outlined above is not feasible.

In most cases, however, it is not even possible to assume that a rationed commodity is replaced by one, or possibly a few, other goods. If, for instance, shoes of all kinds are rationed, what substitute may be found? In such situations it seems to be impossible to envisage a satisfactory method which enables one to measure the effect of rationing in a regular index series.

The important thing to remember, however, is that the introduction of a rationing scheme or a reduction of the ration should in principle always increase the index, whereas action in the reverse direction bring about a reduction of the index. The introduction of rationing or a reduced ration may under no circumstances lead to a reduced index.

Contrary to this principle the Social Welfare Board in 1940 proposed the Swedish cost-of-living index to be computed in such a way that for rationed articles the then fixed base index should be founded upon a comparison between the cost of the rationed quantities at the comparison period and the cost for the original quantities at the base period.[2] This proposal was, however, never accepted.

A group of items disappears

During the war and the immediately ensuring period a number of articles were completely unobtainable on the retail market. As examples, valid for Sweden, the following articles can be mentioned: rice, bananas, goloshes, sheeting, glasses, and coffee-cups.

The problem of allowing for disappearances is, of course, the same as that for rationing; if the ration for an item is zero, this means that the item has disappeared. Except for cases in which a direct

[2] *Förslag* (1940), p. 7.

substitution takes place, it seems impossible to recommend a satisfactory method which can be used in practice for the computation of a regular index series, when whole groups of substitutes have disappeared.

If Laspeyres' formula is used, the missing item may be excluded from the calculations both at the base period and at the reference period. This is, of course, no satisfactory solution, but as this procedure has often been used in such situations, it will be demonstrated that it may lead to very singular results.

If Laspeyres' formula is used and the disappearing group is designated v, and all other groups r, one obtains for time t, when group v is assumed to be still present

$$I_{0t} = \frac{\Sigma p_t^r q_0^r + p_t^v q_0^v}{\Sigma p_0^r q_0^r + p_0^v q_0^v} \tag{5.4}$$

If at time $t + 1$ prices for r are the same and v has disappeared, one obtains

$$I_{0,t+1} = \frac{\Sigma p_t^r q_0^r}{\Sigma p_0^r q_0^r} \tag{5.5}$$

From these two expressions one obtains

$$I_{0,t+1} - I_{0,t} > 0, \text{ if } \frac{\Sigma p_t^r q_0^r}{\Sigma p_0^r q_0^r} > \frac{p_t^v}{p_0^v}, \text{ and}$$

$$I_{0,t+1} - I_{0,t} < 0, \text{ if } \frac{\Sigma p_t^r q_0^r}{\Sigma p_0^r q_0^r} < \frac{p_t^v}{p_0^v}$$

The effect of the exclusion of a group from the index may consequently be that the index increases or decreases. The disappearance of a group should, as shown above, always lead to an increase of the index.

If the group which has disappeared returns and is then reintroduced in the index, this may also affect the index in both directions, even if all prices are the same. The index should in such a situation always show a decrease.

Chapter 6.—The Quality Problem and the Divisia Integral Index

Introduction

According to chapter 1 a 'point-to-point' comparison of two situations is permissible, although it answers only a rather special question. The treatment of the quality problem in chapter 5 has been based upon the assumption that the index is defined in this way, i.e. on the indifference maps and the indifference level of situation 0.

This approach led to the difficulty that goods (varieties), which were only present on the market in situation 1, had to be valued with respect to situation 0, where they were not available. A corresponding difficulty would, of course, have arisen, if the index had been based upon the indifference level in situation 1.

This difficulty will, of course, be the more serious, the more dissimilar the two situations. As pointed out by Marshall, "if we compare distant places or distant times, we find people with different wants, and different means of supplying those wants."[1]

A direct comparison between two situations with quite different articles has no concrete meaning[2] or as Divisia expresses it:

"l'on ne peut établir aucune comparaison sérieuse entre les pouvoirs d'achat d'une monnaie à deux époques où ce qu'on achète avec cette monnaie est très different."[3]

The differences in the available choice of articles are most obvious in the case of geographical comparisons. In the larger cities all round the world all the more important articles may be available, but has a comparison between the cost of living in Stockholm and some remote outpost in the African interior any meaning? The answer to this question will be that as it is not possible in the one place to

[1] *Marshall* (1916), p. 551.
[2] Cf. *Frisch* (1943), p. 38.
[3] *Divisia* (1925), p. 1006.

maintain a standard of living which can be said to be equivalent to that of the other, a cost-of-living comparison based on the indifference level of either place in not feasible.

If the two situations refer to two places or two points in time which are not too distant from each other, the difficulty seems to be less serious. But even then, for certain parts of the consumption, the goods available for retail consumption may be quite different. And the less distant the two situations are from one another, the more accuracy is required of the calculation.

It may be noted that the fact that two situations are contiguous, either temporally or geographically, does not necessarily mean that it is easier to make a direct comparison between them. It may be easier to compare Stockholm with Sydney than to compare it with some country place only 50 miles away. A comparison of the situations before and after a war is definitely more meaningful than a comparison between either period and some situation during the war-period.

This chapter will be devoted to an analysis of the quality problem applying the Divisia integral index which, according to chapter 1, gives a more general solution to the index problem than the indifference defined index based on the indifference level of either situation. As the point of departure we will choose the following hypothetical case, and to start with it will be demonstrated that a treatment of this case from the standpoint of the indifference defined 'point-to-point' comparison leads to an inconsistent result.

A fundamental problem

Assume that in situation 0 there are two varieties *a* and *b* of an article, and that the consumer, to which the index calculation refers, prefers to buy variety *a* only. The indifference map—but not the indifference level—is assumed unaltered. If, now, in situation 1 the consumer has passed over from *a* to *b*, the explanation may be

a) variety *a* has disappeared, or
b) the price relation between *a* and *b* has changed in such a way as to make the transition advantageous (according to the preference maps and the indifference level in situation 0).

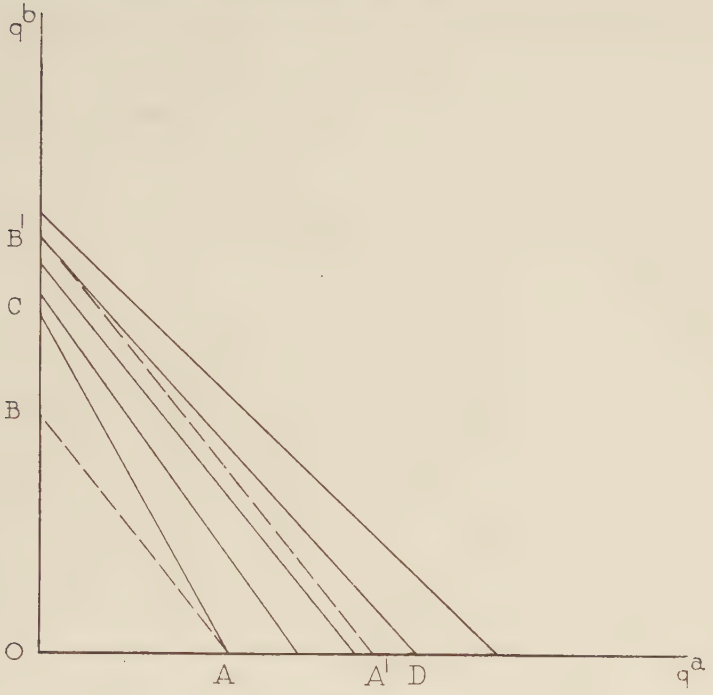

Fig. 6.1

These two cases may be treated by the methods indicated in the preceding chapter. There is, however, also a third possibility, viz.

c) the consumer has been transferred to a new indifference level (prices unchanged; both articles still for sale).

It is also possible that the consumer has *not* passed over from *a* to *b*, in spite of the fact that he should have done so according to the indifference level in situation o and as a consequence of a changed price relation between *a* and *b*.

The transition from *a* to *b* according to Case c) is illustrated in fig. 6.1. In situation o the consumer has an income corresponding to the budget line *AB*, and consequently buys only *a*. In situation 1 he has a higher income, corresponding to the budget line *A'B'*, which is parallel to *AB*, as prices are assumed constant. As on this higher

level the relative valuation of a and b is different—the indifference lines are not parallel—the consumer will now buy variety b only.

As variety a is obtainable in situation 1 and the consumer in situation 0 preferred this variety, the indifference defined index must refer to variety a, i.e. be equal to $\frac{p_1^a}{p_0^a} = 1$. The fact that the consumer has in reality passed over to variety b seems to have no significance. In fact, the index would also be equal to unity, if it had referred to variety b.

So far so good. But let us now split up this case in two different cases. In *Case A* it is assumed that after the situations 0 and 1 ensues a third situation, 2, in which variety a has disappeared from the market. The prices are assumed unaltered. As variety a is without any significance for the consumer from 1 to 2, there seems to be no doubt about the index. If variety b is consumed in both situations 1 and 2, and the price is unaltered, the index must be 1. We have thus obtained $I_{01} = 1$ and $I_{12} = 1$. But it is also seen that if situation 2 is directly compared with situation 0, the result will be $I_{02} = \frac{1}{g} \cdot \frac{p^b}{p^a} > 1$, i.e. $I_{02} \neq I_{01} \cdot I_{12}$.

Let us now alternatively (*Case B*) assume that variety a had disappeared already in situation 1, without the consumer's having been transferred to a new indifference level. We now obtain $I_{01} = \frac{1}{g} \cdot \frac{p^b}{p^a} > 1$. If then from 1 to 2 the increased indifference level is attained, this has no influence on the price index, i.e. $I_{12} = 1$. In this case the direct comparison gives $I_{02} = \frac{1}{g} \cdot \frac{p^b}{p^a} > 1$, and consequently $I_{02} = I_{01} \cdot I_{12}$.

It is obviously difficult to handle the two cases, if the index is defined as an indifference index referring to the indifference level in situation 0. As situation 2 is identical in the two cases, one should obtain $I_{02} = \frac{1}{g} \cdot \frac{p^b}{p^a}$, but in Case A it is not satisfactory to have to assign a price increase to either of the periods, 0 to 1, or 1 to 2.

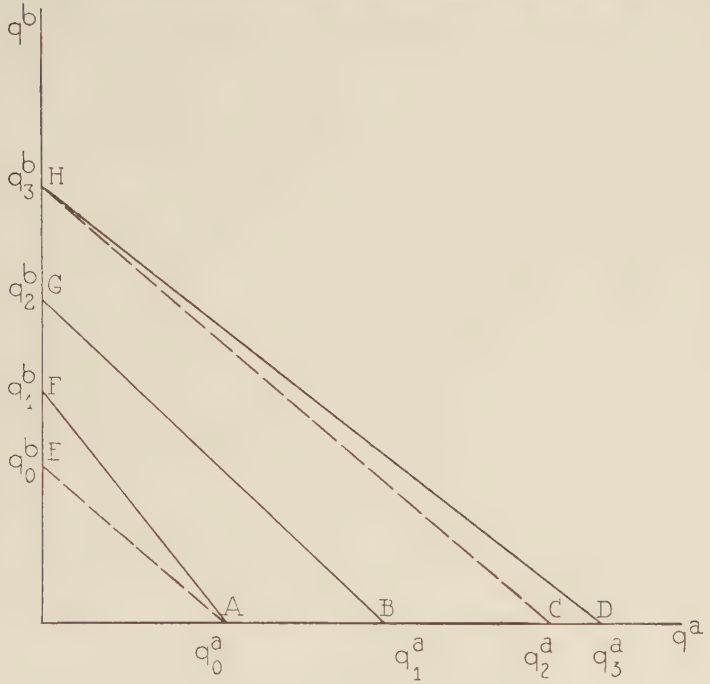

Fig. 6.2

The Divisia approach

If the Divisia index is used, the difficulties disappear. It then becomes clear that the price index *will* be different in the two cases. It seems appropriate to consider this approach at some length.

We will now use diagram 6.2, which is in essence the same as diagram 6.1. In both cases the consumer starts at point *A* and arrives at point *H*. In *Case 1* the consumer follows the q^a-axis from *A* until at point *B*, where the budget line and the indifference line coincide, he jumps to *G*. From then on he is no more interested in quality *a*. He proceeds along the q^b-axis until he reaches *H*.—In *Case 2* variety *a* suddenly disappears. It is assumed that the consumer is compensated, i.e. he receives an income which makes it possible for him to attain point *F*—not more. Afterwards his income increases and finally he reaches point *H*.

The indexes for the two cases have been worked out in the following table.

	From	To	Price index I	Quantity index Q	$I \cdot Q$
Case 1					
	A	B	1	$\frac{p^a q_1^a}{p^a q_0^a}$	$\frac{p^a q_1^a}{p^a q_0^a}$
	B	G	1	$\frac{p^b q_2^b}{p^a q_1^a} = 1$	$\frac{p^b q_2^b}{p^a q_1^a} = 1$
	G	H	1	$\frac{p^b q_3^b}{p^b q_2^b}$	$\frac{p^b q_3^b}{p^b q_2^b}$
	A–B–G–H		1	$\frac{p^b q_3^b}{p^a q_0^a}$	$\frac{p^b q_3^b}{p^a q_0^a}$
Case 2					
	A	F	$\frac{p^b q_1^b}{p^a q_0^a}\left(g = \frac{q_0^a}{q_1^b}\right)$	1	$\frac{p^b q_1^b}{p^a q_0^a}$
	F	H	1	$\frac{p^b q_3^b}{p^b q_1^b}$	$\frac{p^b q_3^b}{p^b q_1^b}$
	A–F–H		$\frac{p^b q_1^b}{p^a q_0^a}$	$\frac{p^b q_3^b}{p^b q_1^b}$	$\frac{p^b q_3^b}{p^a q_0^a}$

There is an obvious parallelism between the cases treated just now and the argument in chapter 1, according to which a different result is obtained if the altered indifference level comes before or after the price change (p. 26). As a matter of fact Case 1 corresponds to the indexes I_1 and Q_0, whereas Case 2 corresponds to the indexes I_0 and Q_1.

There are, of course, other paths from A to H than the two represented by Case 1 and Case 2. Article *a* may thus disappear while the consumer is on the way from A to B, or the consumer may have moved from A towards the origin when *a* disappears.

The general conclusion seems to be that when a transfer from one article to another takes place voluntarily, the Divisia price index cannot be greater than unity. It may be less than unity if *h* is a new article, but the normal interpretation would be that voluntary transfer gives an index equal to unity. When the transfer is enforced,

the index shall be determined from the formula $I = \frac{1}{g} \cdot \frac{p^b}{p^a} > 1$, where the value of g should refer to the valuation of the consumer at the moment when the transference takes place. It will be remembered that g is not a constant but depends upon the indifference level of the consumer. These conclusions confirm the arguments of chapter 3.

Whenever new varieties of different articles are introduced on the market and the consumers gradually and deliberately pass over to them and simultaneously abandon the older articles which consequently disappear—all prices for specified articles are assumed constant—it should consequently not be possible to interpret the transition as an increased Divisia index. Only if the transition from an old to a new variety is enforced, may the index show an increase. But occasions when it is possible to definitely state that a transition is enforced are, as a matter of fact, rare, except during war and rationing. However, the following examples will serve to show that these conclusions lead to results which cannot be accepted.

Some examples

Assume that in situation o there is only one kind of *fountain pen* to be bought. The pen is black and its price is 30 kr. If this pen suddenly disappears from the market and is replaced by another sort of pen, which differs only from the former in that its colour is red and that the price is 39 kr, it seems legitimate to pay no regard to the colour (= quality) change ($g = 1$), and to state that the index is 130.

But assume that the black pens do not disappear suddenly. Instead the new pens are introduced through an advertising campaign, in which the consumers are persuaded that black pens are out of fashion. Whoever wants to keep up to date must buy a red pen. If this campaign is successful, the consumers will 'voluntarily' pass over to the red pens, and the black pens will after a time disappear from the trade. But shall the Divisia index for fountain pens in this case really be unity (or possibly even less)?

In 1948 the "*New Look*" was introduced. Owing to the greater consumption of fabric a New Look mantle cost approximately 10

per cent more than a mantle of the old design. For a while both types were for sale, but it was soon evident that the public preferred the new style. Shall the Divisia index for mantles consequently be judged as unity (or even less)? Husbands paying their wives' dressmaking bills would certainly protest against this result.

Assume that a new kind of *wheat flour*, which is said to be vitaminized, is introduced on the market. The price is 10 per cent higher than that of ordinary flour, and a campaign persuades people to pass over voluntarily to the new kind. The Divisia index will interpret the transition as an index equal to unity or as a price decrease. But if now, after the old kind has disappeared, a scientist discovers that the vitaminisation is of no value whatsoever and that this fact becomes known to the public, what then with the index? From theoretical considerations one may arrive at the conclusion that the Divisia index shall *then* show a price increase. The index will thus show an increase at a moment when both prices and articles are unchanged. What has happened is that the valuation of the article has changed. It would not, presumably, be so easy to convince the public that such an index is of any value.

One of the *vacuum cleaners* used for the Swedish cost-of-living index cost 185 Kr. from 1949 until the summer of 1951. Then it was replaced by a new model costing 210 Kr. The new model had a stronger motor and was thus considered to be more efficient, but it was otherwise identical. Owing to the general price increase in 1951 it seems probable that the price of the old model would have increased if its manufacture had continued. Can it be assumed that the consumer is compensated for the price increase through the improvement of the model?

During the war and the afterwar periods in Sweden *Danish buns* (vinerbröd) were not allowed to contain more than a certain quantity of fat (8 hg fat per liter dough-liquid). In January 1951 when the restrictions were lifted, a new type, containing 12 hg fat per liter, entered the market and soon displaced the older type. The price of the old type was 13 öre and of the new type 15 öre apiece (= 32.5 grammes). Does this transfer represent a price increase or not?

Any practical index computer could easily give numerous other examples of a similar kind. It would also be possible to include here

the different ways in which the goods are offered to the consumer, i.e. different kinds of shops, home delivery or not, time of waiting, etc.

There must be something wrong here. In all the examples given it may be argued that the transfer has taken place voluntarily. However, it is impossible to accept a price index which in all these cases is equal to unity or possibly even less. One answer may be that the assumption of unchanged indifference maps is unrealistic.

Changed indifference maps

Most of the analysis based on indifference maps is founded upon the assumption of unchanged indifference maps. Or, to quote Hicks: "In order to be able to compare the positions of a particular individual in two different situations, we must assume that his wants are the same in the two situations. If this assumption cannot be granted, the question whether he is better off in one situation or in the other loses all economic meaning."[4]

It is true that, as pointed out by Allen[5], for a 'point-to-point' price comparison the indifference maps need not be the same in the two situations. It is possible to base the comparison not only on the indifference level but also on the indifference map of either situation. One is then, however, limited to price comparisons; quantity comparisons can only be undertaken if it is assumed that the indifference maps are the same.

In all the examples given above the transfer from one variety to another may be interpreted as being a consequence of a revision of the indifference maps. This is especially evident with respect to the New Look example. Unless an altered valuation had taken place, the consumers would not have passed over from the old to the new style. Through advertisement campaigns, fashion exhibits etc. the consumers were persuaded that they had to abandon the old style and accept the new, in spite of its being more expensive.

This example is not so extreme as it may seem to be. Modern advertising has a very strong influence on consumers' habits and may often persuade consumers to buy articles which they may soon detect do not give them the satisfaction which they expected. Small quality

[4] *Hicks* (1940), p. 107.
[5] *Allen* (1949).

improvements are often used as sales arguments. A slightly better quality is frequently used as a motive of a considerable price increase (cf. the vacuum cleaner example, above), or expressed in another way, price increases are concealed by insignificant improvements.

It should also be realised that the present day retail trade in industrially well developed countries is so varied that it is impossible for the consumers to have an adequate knowledge of the articles for sale. Measures intended to help the consumers in their choice among the rich variety of articles on the market are often discussed and considered to be of great value. In such connections it is assumed that such measures would tend to give the consumers a better satisfaction than they can now obtain.[6]

Conclusions

The arguments given above as a matter of fact clearly demonstrate the weakness of the indifference map approach to the index problem. The conclusions will be the following.

The valuation of new articles should not only be based on consumers' behaviour, i.e. one should ignore such special and incidental factors which may cause the consumer to pass over from one variety to another. The effect of an advertisement campaign, the tendency of the consumers to buy 'modern' things etc. should consequently be ignored.

Instead the valuation of the different articles (the factor g) should be based upon their function, i.e. to the services which they offer. Factors which are not of real utility should be left out of consideration. This means that it is after all legitimate to use 'objective' criteria for the quality comparisons.

It should be noted that consumption figures give us no extra information. An improved quality may very well result in larger quantities being bought but if the quality change means an increased durability, consumption may also decrease.[7]

Applied to the examples given above, the conclusion will be that the colour of the fountain pens is of no significance, even if it has been possible to persuade the consumers to pass over 'voluntarily'

[6] See, for instance, *Kvalitetsforskning* (1949).
[7] Cf. *Brems* (1951), p. 30 ff.

to this kind of pen. The index shall thus be 130 also in this case. In the same way one should say that the New Look meant that mantles became 10 per cent more expensive, as these mantles rendered the same service as mantles of the older design.

The wheat flour example is harder to treat. The index computer may not have available the scientific instruments necessary in order to ascertain whether the new kind of flour shall be treated as a better quality or not. He may consequently be tempted to believe in the advertisement campaign, although the index should show an increase as soon as the consumer has passed over.

In the vacuum cleaner and the Danish bun examples one may come to the conclusion that part of the price difference is a price increase, whereas the remaining part is compensated for by a quality increase.

In the case of articles which are present on the market at one and the same time in a large number of varieties, the best procedure seems to be that advocated in chapter 4. The specification of the articles for which prices are collected should be reasonably accurate, and the prices obtained for those articles should be accepted.

No general and fixed rules can thus be given. It must be decided from case to case, and from article to article, whether and to what extent a transfer from one variety to another shall be interpreted as a price change. It is unavoidable that such judgments imply to a large extent subjective decisions. The responsibility for sound and as far as possible objective decisions is in the hands of the index computers. As demonstrated in chapter 4, decisions of this kind may have an important effect on the numerical magnitude of the index.

The same principles should be applied to geographical comparisons. A good example of the difficulties is the following one which has been taken from a cost-of-living comparison between New York and Geneva undertaken in the autumn of 1951. This investigation, in which the author took part, aimed at giving a basis for salary comparisons for U. N. officials in the two cities.

The price of ordinary American *cigarettes* was in New York 21 cents for a packet of twenty. The price of the same brands (Chesterfield, Lucky Strike etc.) in Geneva was 1.40 S. frs. = 33 cents (exchange rate 1 $ = 4.28 S. frs.). One may consequently come to the

conclusion that the price ratio for cigarettes for Geneva over New York would be 33:21 = 157 per cent. But in Geneva the American brands were only bought by a minor part of the population. The most common brands were Players No. 6 (U.N. officials) and Gauloises (french-speaking population). The price for both was 0.95 S.frs. = 22 cents. These brands were hardly obtainable in New York, but one may find other European brands which were for sale there, as for instance, Craven A, costing 1.30 S. frs. = 30 cents in Geneva as compared with 30 cents in New York.

Very different conclusions may evidently be arrived at. From the figures given it is possible to state definitely that the price of cigarettes in Geneva as compared with New York must lie somewhere between 157 and 100 per cent, but these limits are so wide that this conclusion is of no great help. As a matter of fact a case can be set up for all values between these limits. The soundest conclusion seems to be that the American brands in New York are comparable with the most common brands in Geneva, i.e. the price difference for Geneva over New York is assumed to be $\frac{22}{21} = 105$ per cent. The 'satisfaction' obtained by smoking Chesterfield is then considered to be the same as that for smoking Players No. 6.

We have thus come to the conclusion that formula (5.1) shall be used whenever a transition to a new variety takes place, and that the factor g shall be determined in the manner stated above. This will have rather remarkable consequences.

Assume that neither quality nor price changes have taken place between two situations. If the consumer has passed over from one variety to another, the price index will then show an increase or a decrease, the only exception being the case when $g = \frac{p^b}{p^a}$.

It may seem difficult to accept a price index increase when the old article is still on the market at an unchanged price. But this is a necessary consequence of the arguments given above.

Some formulas are needed to make possible the computation of an index when the transition is only partial. They will be given in the concluding chapter.

Chapter 7.—The Problem of Aggregation

To which section of the population shall the index refer?

The indifference theoretical analysis of the price index problem in general and of the quality problem in particular as in the previous chapters refers to one single consumer. In principle, though not in practice, it is possible to compute separate price indexes for every individual or at least every household in a population. These separate indexes must be assumed to give different results. This point of view is, however, not of great interest; what one wants to obtain is some kind of average measure of the price development, valid for at least part of the population.

It is often decided that the index shall refer to a certain, more or less sharply defined population group or, to quote Frisch, "we segregate a certain group of individuals, the *definitional group* for the index number in question, for instance, working men's families in the case of a cost of living index."[1] All such families are often postulated to be on the same indifference level and to have the same indifference maps. They must consequently have identical consumption.

This assumption is well in accordance with the way of thinking when the first regular series of cost-of-living index numbers were begun. These series generally referred to the working class only, and often there were in addition other limitations, as for instance to families of a certain composition. The standard of living of the working class at that time was in all countries very low, and any large variations in the use of the money were not possible. At least so it appeared to those who busied themselves with the computation of such series—

[1] *Frisch* (1936), p. 12.

they themselves lived at a considerably higher standard and were apt to consider the conditions of life among working class families as very homogeneous.

But nowadays, at least in industrially more developed countries, it is impossible to find sections of the population, which are homogeneous from the consumption point of view. Even among, for instance, 'urban working class families with two children' great divergences will occur, both with respect to standard of living (indifference levels) and tastes (indifference maps).

Ulmer has expressed the opinion that

> "the area of similarity may be expected to broaden materially as the common environmental factors for groups of people are extended. It is on such grounds as these that it is believed that groups of persons with the same general occupational environment—say, wage-earners—living in the same city or in the same type of city, and receiving approximately the same incomes, possess generally similar patterns of tastes."[2]

According to Ulmer's opinion it is only necessary to consider different incomes, as these are so highly correlated with "many other relevant factors such as cultural background and education." He finally puts the question "How narrowly must an income class be defined, however, in order to insure that the families it comprehends share, at least roughly, common taste patterns?"[3], though without giving any answer.

This opinion does not seem to be very well-founded. A superficial study of consumption patterns is sufficient in order to convince oneself that households of an identical composition and with the same income show very great dissimilarities with regard to the use they make of their money. This is especially obvious for expenditure on alcoholic beverages, tobacco and amusements, but also the buying habits for items in the food, housing and clothing groups etc. vary very much from one household to another. Also with respect to qualities consumption habits may vary considerably: one household may buy razor blades, toilet soap and cigarettes of certain makes, whereas other households prefer different makes.

[2] *Ulmer* (1949), pp. 61–62.
[3] *Ulmer* (1949), p. 62.

A comprehensive index

The question then arises whether it is possible to imagine an index for a group of individuals or households with different indifference maps. If this is possible, it should also be possible to include households of different composition and different income. This problem of aggregation will not be treated at any length here, but some remarks are necessary in order to make it possible to arrive at formulas useful for actual index calculations.

If the indexes for the separate individuals are designated I_0, $I_1, \ldots I_i \ldots$ etc., it is possible to define the comprehensive index in an operationalistic way as the unweighted arithmetic average of these indexes, i.e.

$$\bar{I} = \frac{1}{n} \Sigma I_i \tag{7.1}$$

This point of view can of course also be applied, if Laspeyres' formula is used. Then one obtains

$$\bar{I}_{01} = \frac{1}{n} \sum \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \tag{7.2}$$

where the larger summation sign refers to the individuals and the smaller to the goods.

In practice it is, of course, out of the question to compute index numbers for separate individuals. If Laspeyres' formula is used, this means that instead the following form is employed

$$\bar{I}_{01} = \frac{\Sigma p_1 \bar{q}_0}{\Sigma p_0 \bar{q}_0}, \text{ where } \bar{q}_0 = \frac{1}{n} \Sigma q_0 \tag{7.3}$$

and consequently

$$\bar{I}_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \tag{7.4}$$

But this is the same as a weighted arithmetic average of the separate indexes, where the weights = the expenditure at time $0 = \Sigma p_0 q_0$. For

$$I_{01} = \frac{\sum \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \cdot \Sigma p_0 q_0}{\Sigma p_0 q_0} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \tag{7.5}$$

The implication of this alternative operationalistic way of defining the comprehensive index is that the greater the income of an individual, the more will the changes of the price index for that individual influence the comprehensive index.

This alternative way of defining the comprehensive index has the advantage that if the index shall refer to the whole population of a country, the $\bar{q}_0$-values in formula (7.3) will refer to the whole private consumption of the different articles in the country. These quantities may be easier to determine than the corresponding quantities for some smaller group of the population. The arguments will be the same if another formula is used, e.g. Edgeworth's formula.

Quality changes when the index refers to more than one consumer

To start with, the argument will concern only two consumers, *A* and *B*, and two varieties, *a* and *b*. We will now return to the ideas of chapter 5, i.e. what we want to obtain is a 'point-to-point' comparison based upon the indifference level of situation 0. If now, in this situation, the one consumer buys the one variety and the other consumer the other one, this must imply that their *g*-values are different (g_A and g_B). If this depends upon a difference in the indifference maps or the indifference levels is of no interest in this connection. Four different cases are now possible.

Case 1. In situation 0 consumer *A* buys variety *a* and consumer *B* variety *b*. In situation 1 both varieties are still available and the consumers still buy the same varieties. The comprehensive index will now be

$$I_{01} = \frac{p_1^a q_{0A}^a + p_1^b q_{0B}^b}{p_0^a q_{0A}^a + p_0^b q_{0B}^b} \tag{7.6}$$

This, as a matter of fact, is nothing else than Laspeyres' formula for two items.

Case 2. In situation 0 both consumers buy article *a*. In situation 1 variety *b* has been added to the market and both consumers have transferred from *a* to *b*. The comprehensve index now becomes

$$(7.7) \qquad I_{01} = \frac{p_1^b \frac{q_{0A}^a}{g_A} + p_1^b \frac{q_{0B}^a}{g_B}}{p_0^a q_{0A}^a + p_0^a q_{0B}^a} = \frac{p_1^b}{p_0^a} \cdot \frac{\frac{q_{0A}^a}{g_A} + \frac{q_{0B}^a}{g_B}}{q_{0A}^a + q_{0B}^b}$$

If there are more than two consumers, who have all transferred from a to b one obtains

$$(7.8) \qquad I_{01} = \frac{p_1^b}{p_0^a} \cdot \frac{\sum \frac{q_0^a}{g}}{\Sigma q_0^a}$$

If now one designates

$$(7.9) \qquad \frac{1}{G} = \frac{\sum \frac{q_0^a}{g}}{\Sigma q_0^a}$$

G may be said to represent the average valuation of the consumers of variety b in relation to variety a. The following formula, being as a matter of fact identical with formula (5.1), should thus be used, when all consumers have passed over from variety a to variety b:

$$(7.10) \qquad I_{01} = \frac{1}{G} \frac{p_1^b}{p_0^a}$$

Case 3. In situation 0 both consumers buy only variety a but in situation 1 one of them ($= B$) has transferred to variety b. The index now becomes

$$(7.11) \qquad I_{01} = \frac{p_1^a q_{0.1}^a + \frac{1}{g_B} p_1^b q_{0B}^a}{p_0^a q_{0A}^a + p_0^a q_{0B}^a}$$

If this formula shall give $I_{01} = 1$, i.e. if splicing shall be allowed, the following relation must be fulfilled

$$(7.12) \qquad \frac{1}{g_B} = \frac{p_0^a}{p_1^b} + \frac{q_{0A}^a}{q_{0B}^a} \cdot \frac{p_0^a - p_1^a}{p_1^b}$$

If $p_1^a = p_0^a$, this relation is simplified to

$$(7.13) \qquad g_B = \frac{p_1^b}{p_0^a}$$

If formula (7.11) shall give $I_{01} = \frac{p_1^b}{p_0^a}$ (= the quotient between the prices of both goods), the following relation must be fulfilled

$$\frac{1}{g_B} = 1 + \frac{q_{0A}^a}{q_{0B}^a} \cdot \frac{p_1^b - p_1^a}{p_1^b} \tag{7.14}$$

If formula (7.11) is extended to more than two consumers, one obtains

$$I_{01} = \frac{p_1^a \Sigma q_{0A}^a + p_1^b \sum \frac{1}{g_B} q_{0B}^a}{p_0^a \Sigma q_{0A}^a + p_0^a \Sigma q_{0B}^a} \tag{7.15}$$

Hence, using the relation (7.9):

$$I_{01} = \frac{p_1^a \Sigma q_{0A}^a + p_1^b \frac{1}{G_B} \Sigma q_{0B}^a}{p_0^a (\Sigma q_{0A}^a + \Sigma q_{0B}^a)} \tag{7.16}$$

This is thus the formula which should be used when a new variety has been added to the market.

Case 4. In situation 0 consumer A buys variety a and consumer B variety b, but in situation 1 consumer A has passed over to variety b. The index now becomes

$$I_{01} = \frac{\frac{1}{g_A} p_1^b q_{0A}^a + p_1^b q_{0B}^b}{p_0^a q_{0A}^a + p_0^b q_{0B}^b} \tag{7.17}$$

If this formula which is, as a matter of fact, identical with formula (5.3) shall give $I_{01} = 1$, the following relation must be fulfilled

$$\frac{1}{g_A} = \frac{p_0^a}{p_1^b} + \frac{q_{0B}^b}{q_{0A}^a} \cdot \frac{p_0^b - p_1^b}{p_1^b} \tag{7.18}$$

If $p_0^b = p_1^b$, this becomes

$$g_A = \frac{p_0^b}{p_0^a} \tag{7.19}$$

If in situation 0 consumer A preferred variety a, then $g_A < \frac{p_0^b}{p_0^a}$ and

consequently $I_{01} > 1$ (if $p_0^b = p_1^b$). If then a variety, which one of the consumers bought in situation 0, has disappeared, the index should show an increase, provided that the price of the other variety, which the consumer could pass over to, has kept its price unchanged.

If formula (7.17) shall give $I_{01} = \frac{p_1^b}{p_0^a}$, the following relation must be fulfilled

$$\frac{1}{g_A} = 1 + \frac{q_{0B}^b}{q_{0A}^a} \cdot \frac{p_0^b - p_0^a}{p_0^a} \tag{7.20}$$

If formula (7.17) is extented to more than two consumers, one obtains

$$I_{01} = \frac{p_1^b \frac{1}{G_A} \Sigma q_{0A}^a + p_1^b \Sigma q_{0B}^b}{p_0^a \Sigma q_{0A}^a + p_0^b \Sigma q_{0B}^b} \tag{7.21}$$

where $\frac{1}{G_A} = \frac{\sum \frac{q_{0A}^a}{g_A}}{\Sigma q_{0A}^a}$

This formula shall thus be used when a variety has disappeared.

Partial transition: new article

As mentioned above, formula (7.16) should be used, when a new article has entered the market, i.e.

$$I_{01} = \frac{p_1^a q_{0A}^a + p_1^b \frac{1}{G} q_{0B}^a}{p_0^a (q_{0A}^a + q_{0B}^a)}$$

But here q_{0A}^a and q_{0B}^a cannot in general be observed (separately). However, if $p_0^a = p_1^a$ a reasonable assumption seems to be that

$$\frac{q_{0A}^a}{q_{0A}^a + q_{0B}^a} = \frac{q_{1A}^a}{q_{1A}^a + G q_{1B}^a} \text{ and} \tag{7.22}$$

$$\frac{q_{0B}^a}{q_{0A}^a + q_{0B}^a} = \frac{G q_{1B}^b}{q_{1A}^a + G q_{1B}^b} \tag{7.23}$$

If these values are inserted in (7.16), one obtains

$$I_{01} = \frac{p_1^a q_{1A}^a + p_1^b q_{1B}^b}{p_0^a (q_{1A}^a + G q_{1B}^b)} \tag{7.24}$$

Partial transition: old article

If both varieties *a* and *b* are consumed in both situations, the problem is more complicated. Assume that at time o groups *A* and *B* consume variety *a* (quantity $= \Sigma q_0^a$) and group *C* consumes *b* (quantity $= \Sigma q_0^b$). In situation 1 group *B* has passed over from *a* to *b*. The index should now take the form

$$I_{01} = \frac{p_1^a q_{0A}^a + p_1^b \frac{1}{G} q_{0B}^a + p_1^b q_{0C}^b}{p_0^a (q_{0A}^a + q_{0B}^a) + p_0^b q_{0C}^b} \tag{7.25}$$

If $G = \frac{p_1^b}{p_1^a}$ the formula will give the same result as Laspeyres' formula, i.e. if prices are the same it will give $I_{01} = 1$. But the argument is founded on the assumption that this relation is not fulfilled.

Formula (7.25) then may give $I_{01} \neq 1$, even when prices are constant. As already pointed out in the previous chapter, this is awkward; if both prices and goods for sale are unchanged, it would seem as if the index must also be unchanged, even if changes in the consumption of the different goods have taken place. Formula (7.25) is, however, a necessary consequence of the earlier argument. If one wishes to avoid a sudden change in the index, when a variety finally disappears from the market, after having long ago lost its significance to the consumers, it is necessary that displacements between varieties must influence the index also during a period when the same varieties are for sale and even when all prices are unchanged.

Formula (7.25) cannot be used as it stands. Observations can only give the prices and the quantities q_0^a $(= q_{0A}^a + q_{0B}^a)$ and q_0^b $(= q_{0C}^b)$, but not q_{0A}^a and q_{0B}^a separately. But as q_1^a $(= q_{1A}^a)$ and q_1^b $(= q_{1B}^b + q_{1C}^b)$ are observable, the following relations can be assumed to hold, if $p_0^a = p_1^a$ and $p_0^b = p_1^b$:

$$q_{0A}^a = \frac{q_1^a (q_0^a + G q_0^b)}{q_1^a + G q_1^b} \tag{7.26}$$

$$q_{0B}^{a} = G\frac{q_0^a q_1^b - q_1^a q_0^b}{q_1^a + Gq_1^b} \tag{7.27}$$

If these values are inserted in formula (7.25) one obtains

$$I_{01} = \frac{(p_1^a q_1^a + p_1^b q_1^b)(q_0^a + Gq_0^b)}{(p_0^a q_0^a + p_0^b q_0^b)(q_1^a + Gq_1^b)} \tag{7.28}$$

If $q_0^b = 0$, i.e. if variety b is new in situation 1, this formula simplifies to formula (7.23).

Another interpretation of the formulas

In the formulas given above G refers to the average valuation of variety b relative to variety a among the consumers (all of them or only a certain part). If, instead, the concluding ideas of chapter 6 are applied, G should represent a valuation of the varieties which is based upon their function (see p. 120). This interpretation, which leads to the same formulas, makes it possible to use them in actual practice.

The following example illustrates the procedure.[4] Assume that at a certain point in time ($= 0$) there is only one (plainer) kind of cheese ($= a$) for sale, price 2.55 kr per kg ($= p_0^a$). The consumed quantity is 30 kg per household per year ($= q_0^a$). At a later point in time ($= 1$) a new and better kind of cheese ($= b$) has entered the market. Its price is 3.55 kr per kg ($= p_1^b$). The consumers have partly passed over to the new kind, and one finds that $q_1^a = 20$ kg and $q_1^b = 10$ kg. The price for kind a is unchanged ($p_1^a = p_0^a$).

Assume now that a valuation of the two kinds, based on caloric content etc., gives as result that kind b can be regarded as 30 per cent better than kind a, i.e. that $G = 1.3$. Formula (7.24) now gives

$$I_{01} = \frac{2.55 \cdot 20 + 3.55 \cdot 10}{2.55\,(20 + 1.3 \cdot 10)} = 1.03$$

A complete transition from a to b would have given

$$I_{01} = \frac{1}{1.3} \cdot \frac{3.55}{2.55} = 1.07$$

[4] The example is well in accordance with the real course of events in Sweden in the winter of 1949–50.

If the prices have changed from 0 to 1, formula (7.24) may be used only under the assumption that the price change (for a certain variety) has had no influence on the consumption. Provided that the price change for variety a is not simultaneous with the introduction of the new variety b, the points in time can, at least in principle, be chosen so near each other that the introduction of variety b in the index can be considered separately.

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